Lecture 3: Descartes Theorem and its Generalization (Undergraduate Lecture) 4 p.m., Friday, April 1, 106 Deady

Abstract: It is clear that three pairwise tangent discs on a plane can have arbitrary radii r_1, r_2, r_3. But for a fourth disc, tangent to these three, the radius r_4 must satisfy some equations. This equation was first discovered by René Descartes in the XVII century and impressed many people, even non-mathematicians. It turns out that this equation admits two nice reformulations: in terms of Hermitian 2×2 matrices and in terms of space-like vectors in special relativity.

These reformulations allow not only to give a "natural" proof of Descartes' equation, but also essentially generalize it. The filling of a unit disc by discs with integral boundary curvatures, arising here, leads to several beautiful geometric, group-theoretic, and arithmetic questions which are mostly unsolved.