## Lecture 2: Self-Similar Fractal Sets and Generalized Numerical Systems (Undergraduate Lecture)

**12 p.m., Wednesday, March 30, 229 McKenzie Hall**

Abstract: What is and what can be a numerical system? Usually, to encode a real number we use an infinite sequence $a = (a_1, a_2, a_3, ...)$ where all "digits" $a_i$ take values from some set *A*. For example, we can put *A* = {0, 1} and associate to a sequence *a* the number $\mathrm{val}(a) = \sum_{k>0} a_k / 2^k$ (the standard binary system). If we keep *A* = {0, 1} and replace 2 by some exotic base *b* (e.g. by –2 or by 1 + *i*, the set of possible values val(*a*) can be a fractal.

There are quite different ways to associate a number val(*a*) to a sequence *a*. For instance, continuous fractions $a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cdots}}}$ give such a way.

It is interesting that all these "numerical systems" can be considered as particular cases of a general scheme which we call matrix numerical systems. A new class of function arises when we use these generalized numerical systems. Namely, we can write a number $x$ in one system and then read it in another one. We get a functions $x \mapsto y$, which usually can not be expressed in terms of known elementary functions. As examples, I mention the "question function" of Minkowski and harmonic functions on the Sierpinski gasket.