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AGENCY COSTS IN DYNAMIC ECONOMIC MODELS*

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We consider an overlapping generations economy where capital is produced from bank loans under stochastic constant returns to scale, and subject to idiosyncratic shocks whose realisations are costly to verify. Our formulation differs from earlier work in permitting investment projects to be infinitely divisible and private agency costs to be convex. If there are external economies to financial intermediation, then deviations from steady-state output are negatively correlated with the spread between loan and deposit rates. Moreover, the capital stock correspondence is set-valued, a result consistent with poverty traps, growth cycles, and hump-shaped impulse response functions.

This paper investigates how nonconvexities in financial intermediation, particularly in the cost of collecting private information about borrowers, enrich the operating characteristics of a simple, and relatively tractable, one-sector growth model. The enriched set of equilibria is consistent with a number of phenomena that are hard to reconcile with convex models of economic growth: poverty traps, rank reversals, growth cycles and hump-shaped impulse response functions are all possible outcomes of nonconvex information costs.

Bankers and economists alike have long regarded the credit market as key to understanding economic development and to transmitting cyclical shocks through modern industrial economies. The growth branch of this literature starts with Gurley and Shaw (1967) who note that economic growth is almost universally accompanied by *financial deepening*, that is, by more intensive use of external finance in investment and by a gradual lifting of distortions in the credit market. The cyclical fluctuations branch of the literature focuses on the connection between credit market conditions and business cycles; a key concern here is how the credit market propagates and amplifies external shocks through the entire economy. The general idea dates back at least to Keynes, Fisher, and Friedman and Schwartz who argued that adverse conditions in financial markets may have exacerbated the effects of prewar recessions, including the Great Depression.

Greenwood and Smith (1995) survey work on development. Among recent contributions, we note Bencivenga and Smith (1991) who study the growth effects of financial intermediation in an overlapping generations model with uncertain liquidity needs. Intermediation enhances growth because banks are efficient providers of liquidity which frees individuals from the need to maintain low yielding liquid assets. Similar results are obtained in Greenwood and Jovanovic (1990), where intermediary institutions are shown to arise

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endogenously with costly investment in organisational capital. Higher returns earned on capital promote growth, which in turn enables economies to utilise more efficient and costlier financial structures. In related work, Saint-Paul (1992) explores the interaction between financial markets and the choice of technique. Countries with poorly developed financial markets choose less risky but also less productive technologies; well organised financial markets spread risks better and encourage the adoption of more highly specialised and productive technologies. Acemoglu and Zilibotti (1997*a*) demonstrate how incomplete credit markets lead to non-ergodic growth: the growth path depends on opportunities for diversifying risk which, in turn, are influenced by the initial stock of wealth.

Much business cycle research investigates the informational role played by financial intermediaries. In both Williamson (1987) and Bernanke and Gertler (1989), for instance, the key mechanism is the link between borrower net worth and the 'external finance premium'. Investment projects are indivisible, non-transferable and of fixed size. Producers are often unable to finance the entire project from retained earnings, and must rely in part on external funds advanced by banks and similar financial intermediaries. Borrowers and lenders are however asymmetrically informed about investment outcomes. While borrowers observe these outcomes straightaway, banks are able to do so only by incurring a verification or auditing cost. Since borrowers have incentives to misreport project outcomes and declare bankruptcy, verification will actually occur in some states even though it is costly. To cover agency costs, lenders demand an external finance premium over and above their own cost of capital, which depends inversely on the borrower's net worth and directly on loan size. Since the borrower's net worth tends to be procyclical, the external finance premium is countercyclical, and so are agency costs per unit loan.

Labadie (1996) studies this problem in an exchange economy where projects are of variable size instead of being lumpy, and shows how agency costs can amplify idiosyncratic shocks. Persistence and amplification mechanisms of a similar kind are considered by Azariadis and Smith (1998). In their model capital is financed entirely by credit, and there exists an adverse selection problem in the loan market. The resulting equilibria are indeterminate and can display complicated periodic cycles. The quantitative importance of such an amplification mechanism is an issue that concerns Carlstrom and Fuerst (1997). The Bernanke-Gertler setup is modelled in an infinite horizon production economy and calibrated to fit some of the main features of the United States economy. Significantly, the model naturally delivers a hump-shaped investment curve as observed in the data because of the procyclicality of borrower net worth. However, to generate persistence in the data, the authors have to rely on correlated aggregate productivity shocks.

Both persistent growth and external shock amplification are ultimately connected with some type of nonconvexity. The financial intermediation literature delivers these properties by assuming indivisible investment projects or fixed costs of intermediation. It turns out that nonconvex costs in general, and decreasing unit costs of intermediation in particular, are instrumental in delivering a rich pattern of equilibria which includes poverty traps, growth cycles, rank reversals and many other dynamic properties that are puzzles for convex growth models.

In this paper we assume that all individual costs of intermediation are convex but allow for external economies of scale. We embed a credit market with asymmetrically informed borrowers and lenders into a two period overlapping generations model. The agency cost we focus on is the cost of state verification. We distinguish between two types of goods, a capital good and a consumption good. Final goods are produced from labour and capital using a standard neoclassical production function, but capital formation is financed by credit. Specifically, capital is produced from bank loans using a constant-returns technology subject to idiosyncratic shocks; the realised values of these shocks can be verified by financial intermediaries only at a cost. Financial intermediaries deal with the moral hazard problem associated with asymmetric information, and diversify idiosyncratic project risks. As a result depositors are paid a sure deposit rate while borrowers receive (standard) debt contracts which verify project outcomes in some states.

When unit costs of state verification are constant, our model has a unique positive, asymptotically stable steady state. Higher agency costs imply a lower steady-state capital stock, but cannot by themselves generate multiple equilibria, propagate external shocks or cause cyclical changes in the external finance premium.

We next explore decreasing unit agency costs at the aggregate level because indivisibilities and nonconvexities play a big role in the credit market models of Bernanke and Gertler (1989) and Williamson (1987). The key general equilibrium implications of nonconvex verification costs are a negative correlation between interest rate differentials and deviations from steady state output, and the possible existence of multiple equilibria. In particular, the equilibrium capital stock correspondence, or 'policy function', may be set-valued, a property which potentially acts as a propagation and amplification mechanism for technology and other external shocks. Simple numerical calculations show that the strength of decreasing agency costs necessary for such set-valued dynamics is consistent with existing evidence on the cyclical behaviour of the interest rate spread between loan and deposit rates.

The paper is organised as follows. In the next section we explain the structure of the model, the problems solved by agents, describe the optimum loan contract and define general equilibrium. Section 2 considers the general equilibrium properties with constant agency costs. In Section 3, we examine the consequences of external economies in agency costs. Section 4 provides some numerical examples which connect set-valued dynamics in the model with the cyclical behaviour of the interest rate gap. Section 5 discusses amplification, growth cycles and other complex dynamic properties associated with nonconvex costs.

1. Structure of the Model

1.1. *Assumptions*

Consider a two-period overlapping generations economy with constant population. Each generation has a continuum of agents with unit mass and consists of two types of agents, working households and investment goods producers.

A fraction $\lambda \in [0, 1]$ of the population is households. They do not own any capital or final goods, but are endowed with 1 unit of labour time in youth, which is supplied inelastically. Households consume in both periods of life, evaluating consumption vectors by the logarithmic utility function:

$$U^H(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t.$$

Here a superscript denotes the generation, and a subscript denotes calendar time. More general utility functions are easy to accommodate if dated consumption goods are normal and are gross substitutes. Young households deposit their savings with banks in period t and are paid a sure return R_{t+1}^D the following period on each unit deposited. The initial old generation of households have an aggregate capital endowment of $K_0 > 0$.

The remaining fraction of the population are investment goods producers or investors. They are risk-neutral and consume only in old age. Investors have no endowment of time or goods. Each of them owns an investment technology. In period t, a young investor i can borrow b_t units and convert it into capital in period $t+1$ according to the stochastic constant returns technology:

$$k_{t+1} = \theta^i b_t.$$

θ^i is a privately observed stochastic shock distributed independently and identically across investors, with mean one and a cumulative distribution function $H(\theta^i)$ on the bounded support $\Theta \equiv [\underline{\theta}, \overline{\theta}]$. Each investor observes her idiosyncratic shock soon after borrowing (in the same period), but the capital is produced only at the beginning of the following period. This capital is rented out to firms; consumption and loan repayments are then made out of rental income. Note that our assumption that investors start out without any wealth differs from Bernanke and Gertler (1989), where each investor self-finances part of her project with a goods endowment received in youth.

As in the standard overlapping generations model, final goods are produced by firms[1] using labour supplied by young households and capital rented from investors, according to a constant returns to scale production function $F(K, N)$. Markets for labour and capital are perfectly competitive so that factors of production are paid their marginal products. To simplify the analysis we assume that capital depreciates fully. This allows us to ignore sales of undepreciated capital to younger agents.

Borrowing and lending are carried out through financial intermediaries called banks. In the presence of credit market frictions and the possibility of

[1] We have separated the acts of converting loans into capital, and then using it to produce final goods, purely for convenience. Results would be unchanged if we allowed each two period lived investor/entrepreneur also to produce the final good.

an *ex post* moral hazard problem, Williamson (1986) shows that financial intermediation is an optimal arrangement for monitoring borrowers. The bank is able to exploit the law of large numbers by interacting with many borrowers and depositors. Hence it is able predict with certainty the fraction of investments that have bad outcomes, and guarantee a sure return to its depositors. Banks enter into contracts with investors, taking as given the return that has to be paid on deposits. However, because of private information, they are able to verify the project outcomes at a cost. The details of financial contracts are discussed below.

1.2. *Financial Intermediation and Loan Contracts*

The optimal loan contract between banks and borrowers is obtained as a solution to a principal-agent problem. Banks accept deposits from young households promising to pay, at time $t+1$, an amount of R^D_{t+1} for each unit deposited at t. Lending to investors is governed by the terms of a loan contract. When offering these loan contracts, banks act as Nash competitors who maximise profits subject to the contract being incentive compatible as well as satisfying an individual rationality (or participation) constraint for each potential borrower. With positive costs of state verification and deterministic monitoring, Gale and Hellwig (1985) and Williamson (1986) show that the optimal loan contract is a standard debt contract of the form $\delta = (b,\ x,\ R^L) \in R^3_+$. This triple specifies the loan size b, a critical value $x \in \Theta$ for the idiosyncratic state θ, and the gross yield R^L owed to the bank on each unit borrowed by the investor.

All incentive compatible contracts must verify in some states, otherwise borrowers would always default. Accordingly, the contract specifies a critical state x below which verification occurs for sure. In particular, if the realised idiosyncratic shock is less than x, the investor is unable to repay bR^L and declares bankruptcy. The bank verifies the state, paying a proportional cost of γ units of capital per unit loan,[2] brings the bankrupt project to completion, and recovers an amount $X(\theta)$ from it. In other words, we assume that once a project is declared bankrupt, the bank takes over and rents out whatever capital is produced. Therefore, the recovery amount $X(\theta)$ is simply the rental income from the project, $\rho\theta b$, where ρ is the rental rate for capital.

On the other hand, if the realised idiosyncratic shock is above x, the investor is able to pay back bR^L. Incentive compatibility requires that the loan rate, R^L, be independent of the realisation of the private shock, and that the payoff function be continuous in θ, so that $R^L b = X(x) = \rho x b$. Accordingly the contract can be written as the triple $\delta = (b,\ x,\ \rho x)$.

An investor's expected payoff from a contract δ is

[2] Unlike Townsend (1979) or Bernanke and Gertler (1989), we assume that bank monitoring is deterministic instead of stochastic. In this we follow Boyd and Smith (1994) who observe that gains from stochastic monitoring over deterministic monitoring are not large and that actual debt contracts are seldom as complicated as the ones with stochastic monitoring.

$$U(\delta) = b\rho[1 - \mu(x)], \tag{1}$$

where $\mu(x) = \mathrm{E}\min(\theta, x) = x[1 - H(x)] + \int_{\underline{\theta}}^{x} \theta \, dH$. Similarly, if $\gamma \in [0, \underline{\theta}]$ is the proportional cost of verification, expected bank profit is,

$$\Pi(\delta, q) \equiv \rho b[M(x, \gamma) - q], \tag{2}$$

where, $q \equiv R^D/\rho$ and

$$M(x, \gamma) \equiv x[1 - H(x)] + \int_{\underline{\theta}}^{x} (\theta - \gamma) \, dH. \tag{3}$$

We assume free entry into banking, so that equilibrium bank profits must be zero.

THEOREM 1 *Given* (ρ, γ), *and the expected investor payoff* U_0, *the optimum loan contract* $\hat{\delta} = (\hat{b}, \hat{x}, \hat{R}^{\mathrm{L}})$ *satisfies the following conditions:*

(i) $\hat{x} = \arg\max_{x \in \Theta} M(x, \gamma)$
(ii) $\hat{R}^L = \rho\hat{x}$, *and*
(iii) $\hat{b} = U_0/\rho\{1 - \mu[\hat{x}(\gamma)]\}$.

Details of the loan contract are provided in the Appendix. Note that the average loan size is indeterminate here, and has to be pinned down by the total flow of funds into banks. Investor utility U_0 is determined endogenously in general equilibrium. The current capital stock determines wages and hence, total deposits. Deposits, in turn, influence the volume of loans and next period's capital stock.

1.3. *General Equilibrium*

By the law of large numbers, the return per unit loan, net of auditing costs, is $\rho M(x, \gamma)$ which, by the zero profit condition, equals the cost of loanable funds $\rho q \equiv R^D$. Hence, the bank's balance sheet satisfies (4)

$$D_t = L_t + \gamma L_t H(x_{t+1})$$

$$\Rightarrow L_t = \frac{D_t}{1 + \gamma H(x_{t+1})}, \tag{4}$$

where $x_{t+1} = \hat{x}(\gamma)$. Equation (4) says that the inflow of funds into banks equals the outflow of funds plus the reserves for agency costs to be incurred on current loans. The time profile of bank borrowing and lending is illustrated in Fig. 1. We assume that banks hold reserves for agency costs to be incurred later in period t, after deposits are made. Remaining funds are then loaned out to young investors. Investors observe their shocks soon thereafter, and the ones who realise particularly bad shocks (less than x) declare bankruptcy at once. The bank takes over these bankrupt projects, brings them to completion, and recovers whatever it can from them. The assumption that provision for agency costs is made in advance is an important one – it makes current loans depend upon expectations of future events.

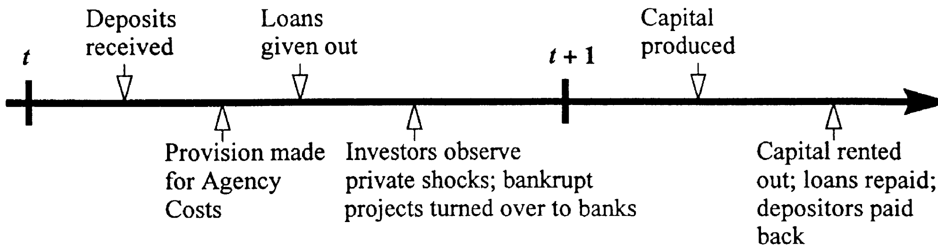


Fig. 1. *Timeline of Bank Operations*

Total deposits D_t are simply total savings by all young households, $sw(k_t)$, where $s \equiv \beta/(1+\beta)$ is the savings rate and $w(k)$ is the wage rate expressed as a function of the current capital-labour ratio. Therefore total loans are

$$L_t = \frac{sw(k_t)}{1+\gamma H[\hat{x}(\gamma)]}.$$

Since $\mathrm{E}(\theta) = 1$, the law of large numbers implies that the capital stock in period $t+1$ is equal to loans made in period t, $k_{t+1} = L_t$.

Dynamic equilibria in this economy then satisfy the following first order difference equation in the capital-labour ratio:

$$k_{t+1} = \frac{sw(k_t)}{1+\gamma H[\hat{x}(\gamma)]}. \tag{5}$$

This equation differs from the standard overlapping generations model only in the denominator of the right-hand side, which would have been equal to one under public information and with constant population. Here, the denominator exceeds one by an expression that equals the unit cost of state verification times the fraction, ϕ, of projects actually verified, where

$$\phi(\gamma) = H[\hat{x}(\gamma)]. \tag{6}$$

For future reference we define the interest rate differential, i.e., the spread between the loan and deposit rates, as

$$\Delta_t \equiv R^L_{t+1}/R^D_{t+1} = \frac{\hat{x}(\gamma)}{M[\hat{x}(\gamma), \gamma]}. \tag{7}$$

This ratio is strictly greater than one, and accords well with evidence from the United States data on the T-Bill – prime rate spread as shown in Fig. 2.

2. Growth with Constant Agency Costs

Consider first the case where the proportional cost γ is a constant, as in Bernanke and Gertler (1989) and Labadie (1996), independent of economic activity.

The phase diagram for this economy is depicted in Fig. 3. When $\gamma = 0$, all information is public since project returns are costlessly observable and (5) is the standard growth model with an asymptotically stable positive steady state

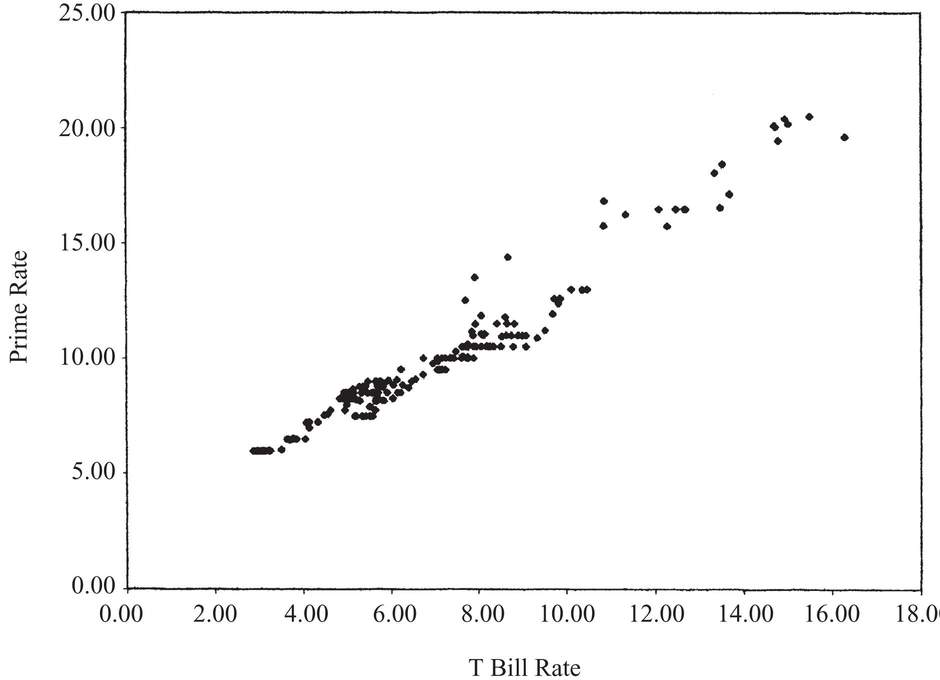


Fig. 2. *T-Bill – Prime Rate Spread in the U.S., 1980–97*

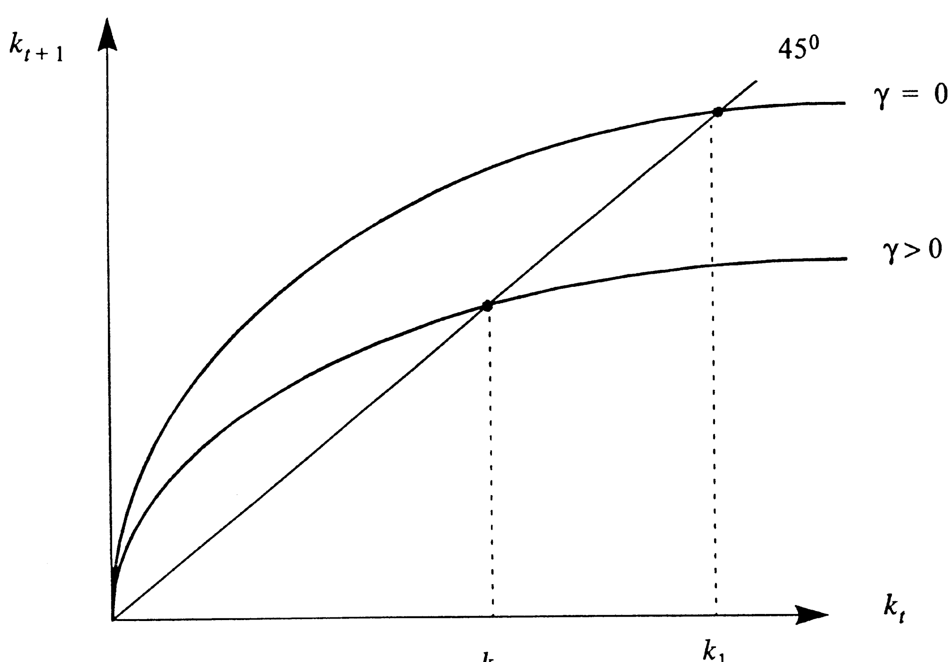


Fig. 3. *Dynamics with Constant Agency Costs*

k_1. Positive costs of verification depress economic activity since agency costs create a wedge between deposits and loans. As these costs increase, ϕ goes down; fewer projects are verified but total agency costs still rise in response to the higher unit cost γ. Each value of the current capital stock now corresponds to a lower value for the future capital stock; the phase diagram in Fig. 3 shifts down. However the economy still possesses a single positive steady state, $k_2 (< k_1)$, which is again asymptotically stable.

The cost parameter γ can potentially capture degrees of credit market imperfection which vary among developed and less developed countries. In particular, developed countries with better systems of risk verification and information pooling would be expected to have lower costs.[3] On that basis, the model predicts that cross-country differences in GDP per capita are explained by differentials in the parameter γ. But how much cross-country differences are accounted for by such costs?

The following illustrative example should help. Suppose θ is uniform on $[1-\varepsilon, 1+\varepsilon]$, and the production function is Cobb-Douglas so that $f(k) = k^{\alpha}$. Then (5) becomes:

$$k_{t+1} = \frac{sw(k_t)}{1+\gamma\phi},$$

where the wage rate is $w(k_t) = (1-\alpha)k_t^{\alpha}$ and the fraction of bankrupt projects is $\phi = (2\varepsilon - \gamma)/(2\varepsilon)$. To see this, start with the likelihood function $\mathscr{L}(x) = 1/(1+\varepsilon - x)$, recall the first-order condition $\gamma\mathscr{L}(x) = 1$ for determining the critical value $\hat{x} = 1 + \varepsilon - \gamma$, and use the definition of ϕ in (6). Steady state output as a function of agency costs is:

$$\overline{y}(\gamma\phi) = [s(1-\alpha)/(1+\gamma\phi)]^{\alpha/(1-\alpha)}.$$

Therefore the steady state output ratio between a country without any credit market friction and one with positive agency costs is:

$$r(\gamma\phi) \equiv \overline{y}(0)/\overline{y}(\gamma\phi) = (1+\gamma\phi)^{\alpha/(1-\alpha)} \approx 1 + \alpha\gamma\phi/(1-\alpha),$$

for small enough costs ($|\gamma\phi| \ll 1$).

For instance, if the capital share of output is $\alpha = 1/3$, parameter values $\gamma = 1/2$ and $\phi = 1/4$ imply a GDP discrepancy of about 6% in steady state output. For more realistic parameter values, $\gamma = 1/4$ and $\phi = 1/10$, the steady state output discrepancy is about 1.25%. In severe cases of inefficient government banks, which typically waste large resources in bad loans, take $\gamma = 2/3$ and $\phi = 1/2$ to obtain an output discrepancy of 15%. If we wish to include the adverse effect of credit market conditions on the accumulation of human capital, then it is fair to interpret the capital stock as inclusive of skilled labour. The parameter α in the production function should then be set at $\alpha = 0.6$ to 0.8 to conform with the estimates of Mankiw *et al.* (1992). In that case, an

[3] See World Bank (1989) for evidence that less developed economies have higher per unit bankruptcy costs.

upper bound on the output discrepancy may be computed for $\gamma = 2/3$, $\phi = 1/3$ and $\alpha = 0.8$ to be 4/3 or 133%.

Contrary to the expectations of early researchers like Gurley and Shaw, none of these parameter values can possibly account for much of the massive international differences in per capita GDP levels that we observe. Furthermore, whereas in the data the interest rate gap Δ is countercyclical and declines with growth (financial deepening), here it is independent of economic activity since γ is a constant.

A more promising explanation of cross-country differences in GDP is the existence of multiple, asymptotically stable steady states. Financial market frictions of the type we explore here are able to generate multiple equilibria as long as the economy exhibits some form of increasing returns to scale. For example, increasing returns or scale economies may occur in the banking sector itself. As the volume of financial intermediation expands, banks may become more efficient intermediaries between final borrowers and lenders.

3. Growth with Decreasing Agency Costs

In this section we explore the implications of external increasing returns to scale in financial intermediation. Nonconvexities of this type are at the heart of the propagation mechanisms studied by Bernanke, Gertler and Williamson who typically assume that investment projects have fixed size. This assumption prevents a borrower from investing more in the project when his net worth increases. We assume, in what follows, that each bank's cost of state verification per unit loan is a decreasing function of activity at the industry level; no individual bank can influence unit costs on its own.[4] One motivation for this assumption is the observation by Acemoglu and Zilibotti (1997*b*) that information about an activity is a byproduct of production, learned by doing. For example, larger economies typically have access to well developed credit standards enforced by rating agencies whose services are available to all lending institutions – therefore costs of verification would be expected to decrease with the volume of credit market transactions as measured by total deposits or loans.[5] As we show below, decreasing costs generate interesting dynamics in the model: poverty traps and set-valued dynamics are both possible depending on how costs are affected by aggregate economic activity.

Let $A(\gamma) \equiv \gamma H[\hat{x}(\gamma)]$ be the unit agency cost as a function of γ, where $\hat{x}$ is defined from the first-order condition $\gamma \mathscr{L}(\hat{x}) = 1$. We assume that

$$1 - \gamma h(x) \left[1 + \frac{h(x)}{\gamma h'(x) + h(x)} \right] > 0$$

at $x = \hat{x}(\gamma)$, so that the function A is increasing for $\gamma \in [0, \underline{\theta}]$.

[4] Williamson (1986) and others assume decreasing private monitoring costs and use an upper bound on individual loans to prevent banks from excessively consolidating their portfolios.

[5] In their study of the trucking industry, Guffey and Moore (1991) point out that there is some evidence of scale economies in bankruptcy costs – larger firms seem to have lower direct costs of bankruptcy.

Since provisions for agency costs are made before banks actually give out loans, a reasonable supposition is that they depend on the aggregate volume of current loans. This assumption connects unit agency costs with expectations of *future* business conditions. Specifically, suppose that

$$\gamma_t = \gamma(L_t) = \gamma(k_{t+1}),$$

where γ is a decreasing function of aggregate loans (hence of future capital stock), which takes on a positive value γ_0 at zero, and goes to zero as $L \to \infty$. Let us define the unit agency cost function as $c_L(k) \equiv A[\gamma(k)]$, a decreasing function, which satisfies:

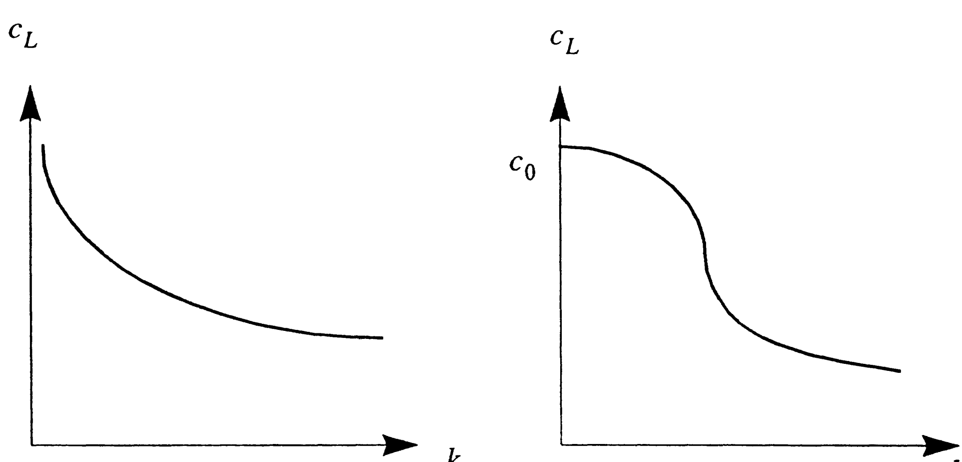
1. $c_L(0) = A(\gamma_0) \equiv c_0 > 0$,
2. $c_L(\infty) = 0$.

Equilibrium sequences are now described by

$$sw(k_t) = G(k_{t+1}) \equiv k_{t+1}[1 + c_L(k_{t+1})]. \tag{8}$$

In this case G is not necessarily monotonically increasing, depending upon how fast agency costs decrease. Fig. 4 illustrates two alternative cost functions. In Fig. 4*a*, costs decline steeply ($c_0 = +\infty$) whereas the fall is initially more gradual in Fig. 4*b*, and then very sharp. Both cost functions correspond to the general shape for G in Fig. 5; the function G is monotone increasing except in the interval $[k_1, k_2]$.

Define k^1, k^2 such that $sw(k^1) = G_{\min}$ and $sw(k^2) = G_{\max}$. Then the forward map described by (8) is single-valued outside the interval $[k^1, k^2]$, set-valued inside that interval, and undergoes bifurcations at k^1 and at k^2. For each value of the present capital stock $k_t \in [k^1, k^2]$ there exist three values of the future capital stock k_{t+1}, that solve (8). These three values define three branches in the capital stock correspondence: the two extreme branches are increasing,



(*a*) (*b*)

Fig. 4. *Two Alternative Cost Functions*

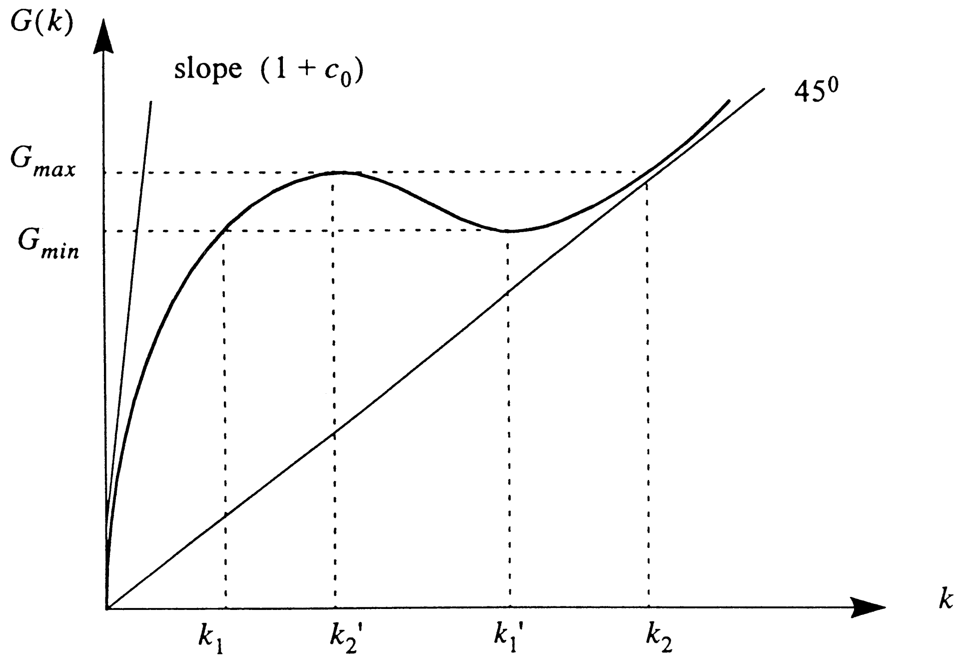


Fig. 5. *Shape of the G function*

the middle branch is decreasing, provided agency costs fall sufficiently steeply inside the interval $[k^1, k^2]$.

Up to three steady states are possible here, one for each branch. While the two extreme steady states k_L and k_H are asymptotically stable, the stability properties of the middle one, k_M, are not obvious. It could be stable or unstable, admitting periodic cycles in its neighbourhood. But the most important feature of the model is that, with forward looking behaviour, it may generate set-valued dynamics. This is similar to what Azariadis and Smith (1998) obtain for an adverse selection economy. The economic intuition underlying these results is similar. Our economy can find itself in any of the three branches shown in Fig. 6. The lowest branch obtains when banks rationally expect future loan activity to be low and unit agency costs to be high. This expectation typically means a high loan rate and a relatively low deposit rate, that is, both low saving and low investment, exactly as banks were anticipating. On the other hand, the high branch of the time correspondence will occur when banks rationally expect loan activity to be high and the interest rate spread to be relatively small.

4. Set-valued Dynamics with Decreasing Costs: A Numerical Exercise

How strong must decreasing agency costs be in order to obtain the set-valued capital stock correspondence of Fig. 6? Does this cost behaviour accord with what we know about the cyclical behaviour of interest rate differentials?

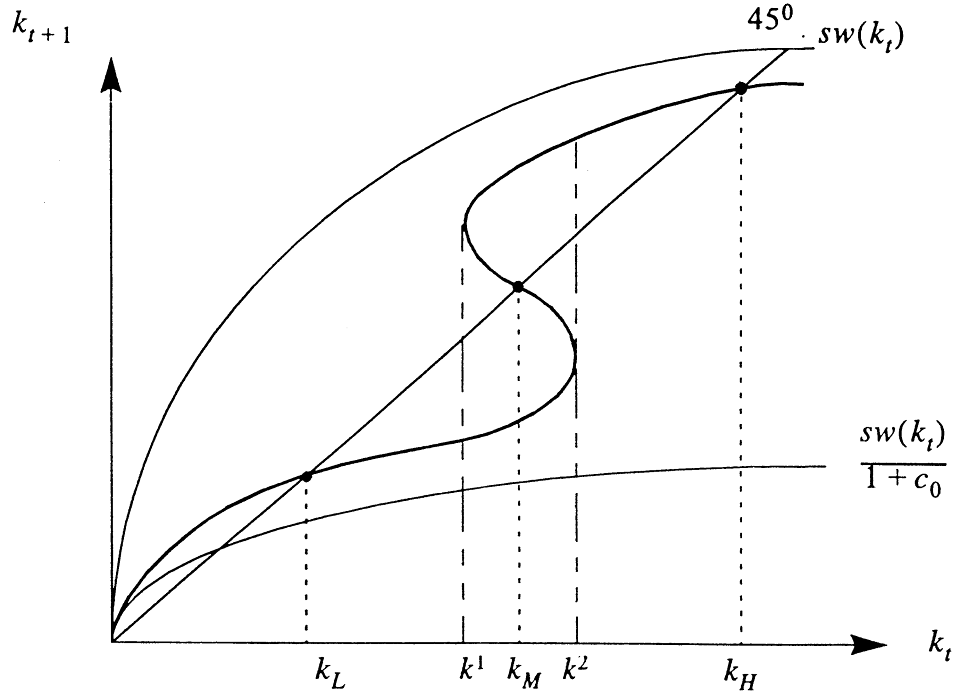


Fig. 6. *Set-valued Dynamics with Decreasing Agency Costs*

Annual growth rates of per capita GDP vary from -2% to $+4\%$ in a typical postwar U.S. business cycle while interest rate spreads change countercyclically by as much as 3 percentage points and lead the business cycle.[6] To see if this strong correlation between changes in output and interest rate gaps is consistent with set-valued dynamics, we return to the parametric example of Section 2.

Assume again that θ is uniformly distributed over $[1-\varepsilon, 1+\varepsilon]$ with the probability density function $h(\theta) = 1/2\varepsilon$. The interest rate differential is $\Delta = \hat{x}(\gamma)/M[\hat{x}(\gamma), \gamma]$, where $M(x, \gamma)$ is defined in (3). For the uniform distribution, it is easy to show, as in Section 2, that $\hat{x}(\gamma) = 1 + \varepsilon - \gamma$ and hence,

$$\Delta = \frac{4\varepsilon(1+\varepsilon-\gamma)}{4\varepsilon(1-\gamma)+\gamma^2} = \frac{1+\varepsilon-\gamma}{1-\gamma+\gamma^2/4\varepsilon}, \tag{9}$$

which turns out to be an increasing function of the unit agency cost γ for all $\gamma \in [0, \varepsilon]$. Furthermore, unit agency costs are

$$c_L(k) = A[\gamma(k)] = \gamma H[\hat{x}(\gamma)] = \gamma - \gamma^2/2\varepsilon. \tag{10}$$

Equation (8) defines a set-valued map if, and only if, $G(k)$ is a decreasing function for some k, that is, iff, for some k

[6] See Stock and Watson (1989), and Bernanke and Gertler (1995) on these issues.

$$G'(k) = 1 + \gamma(k) - \frac{[\gamma(k)]^2}{2\varepsilon} + k\left[1 - \frac{\gamma(k)}{\varepsilon}\right]\gamma'(k) < 0. \tag{11}$$

From (9), (10) and (11) we obtain

$$G'(k) = 1 + \gamma\left(1 - \frac{\gamma}{2\varepsilon}\right) + \eta_\Delta\left(1 - \frac{\gamma}{\varepsilon}\right)\frac{1 + \varepsilon - \gamma}{1 - (1 - \gamma/2\varepsilon)\Delta}, \tag{12}$$

where η_Δ is the absolute elasticity of Δ with respect to k.

Setting $\varepsilon = 1/2$ and $\gamma \approx 0.1$ means that the best project is three times as productive as the worst, and that unit agency costs correspond to about one-tenth the value of a typical loan.[7] These numbers imply that $\Delta = 14/9$, and (12) reduces to

$$G'(k) = 1.09 - 2.8\eta_\Delta < 0 \text{ iff } \eta_\Delta > 0.39.$$

For these parameter values, complex dynamics is possible if a 1% increase in the capital stock (which typically means an increase in per capita GDP of about 0.33%) raises the interest rate gap by at least $(0.39)(14/9) \approx 0.62$ percentage points compounded over the lifespan of a generation. In annual terms, this change is less than 0.2 percentage points. The implied output elasticity of the interest rate gap is well within the range of empirical estimates we discussed earlier; in those we have roughly a 2% increase in GDP following each 1% decline in the interest rate spread.

Condition (11) is not hard to satisfy for reasonable specifications of the γ function. Suppose that agency costs decrease in the capital-labour ratio according to the piecewise linear schedule:

$$\gamma(k) = \begin{cases} \gamma_0 & \text{if } k \in [0,\ k_0], \\ \gamma_0\left(1 - \dfrac{k - k_0}{k_1 - k_0}\right) & \text{if } k \in [k_0,\ k_1], \\ 0 & \text{if } k \in [k_1,\ \infty), \end{cases} \tag{13}$$

where $[k_0,\ k_1]$ is the range within which unit agency costs decrease from $\gamma_0 > 0$ to zero as the capital stock increases.

Equilibrium is then described by the following difference equation:

$$k_t = \left[\frac{G(k_{t+1})}{sA(1-\alpha)}\right]^{1/\alpha} \tag{14}$$

where $G(k_{t+1})$ is defined by (8) and (9), and the production function is $f(k) = Ak^\alpha$.

Figs. 7*a* and *b* display the phase-map for two different parameterisations of k_0 and k_1, and hence the range over which agency costs decline. In both diagrams, we have chosen the parameters $s = \frac{1}{2}$ and $A = \frac{2}{3}$ and $\gamma_0 = \varepsilon = \frac{1}{2}$. For a sufficiently steep fall in costs of verification ($k_0 = 0.085,\ k_1 = 0.095$), the phase map is set valued with three positive steady states and an interest rate gap that

[7] In Altman's (1984) study of 19 industrial firms for the period 1970–8, direct and indirect bankruptcy costs typically range from 11% to 17% of firm value upto three years prior to bankruptcy.

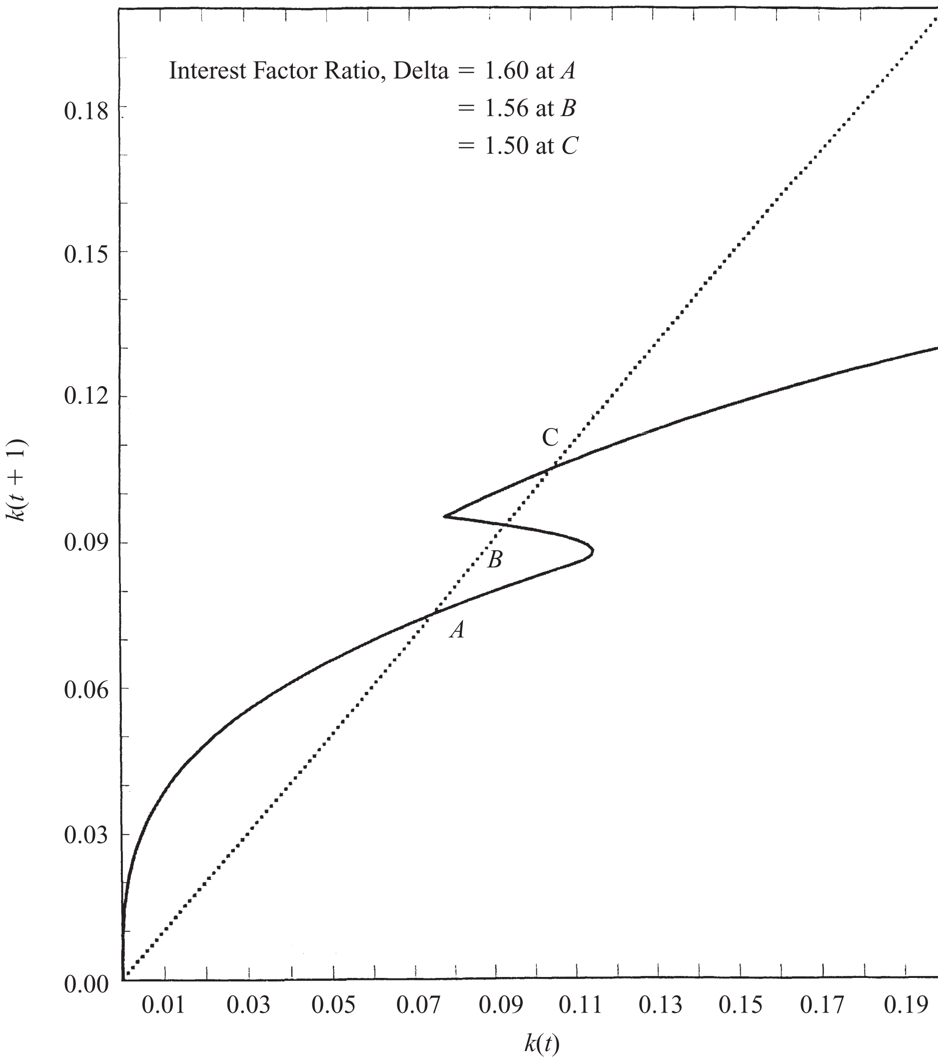


Fig. 7 *(a). Phase Map for* $k_0 = 0.085$ *and* $k_1 = 0.095$

changes ten percentage points between the lowest and the highest steady states (Fig. 7*a*). On the other hand, if costs fall more gently ($k_0 = 0.060$, $k_1 = 0.120$), the phase map is monotonic and a unique positive steady-state exists (Fig. 7*b*).

5. The Economic Implications of Decreasing Agency Costs

This section outlines briefly how an economy with the equilibrium correspondence of Fig. 7*a* could contribue to explaining some aspects of complex dynamic phenomena in economic development as well as in business cycles.

Among the former we count poverty traps, growth cycles and rank-order

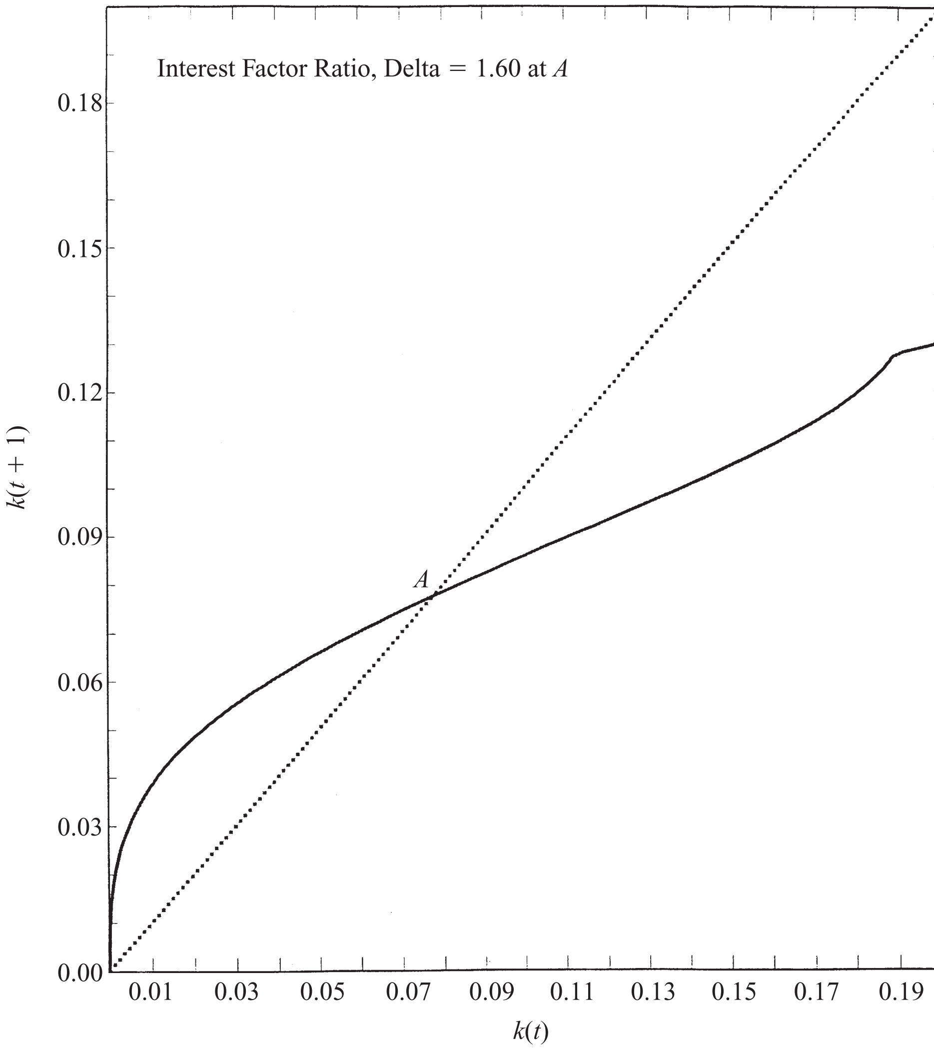


Fig. 7*(b). Phase Map for* $k_0 = 0.060$ *and* $k_1 = 0.120$

reversals; the latter include output trend reversal and amplification in response to temporary external shocks.[8] To grasp what is at stake, we return to Fig. 7*a* and interpret the state variable as the deviation of the capital stock from its trend rate of growth. We think of trend growth simply as reflecting an exogenous rate, *g*, of labour-augmenting technical progress which may correspond to the highest steady state at point *C*. Poverty traps in this setting are

[8] See Azariadis (1996), Boldrin and Rustichini (1994) and Matsuyama (1995) for complex dynamic models relevant to economic development, and Cogley and Nason (1995) for an analysis of the dynamics of United States macroeconomic time series at business cycle frequencies.

equilibria that would start far below the trend rate g and converge to a growth rate less than g, corresponding to the steady state at point A. Section 3 outlines one potential reason for persistently low growth: low-income economies are inherently low-information economies with high unit costs of financial intermediation, and big gaps separating deposit from loan rates.

A similar argument applies to growth cycles of the type studied in Evans *et al.* (1998). These are equilibria that alternate between two or more stable branches of the equilibrium correspondence in Fig. 7*a* with the growth rate switching between a high regime near C, a medium one near B, and a low one near A.

Dynamic equilibria that alternate or switch among several regimes may also help explain how external disturbances become amplified in business cycles. This phenomenon is captured in Cogley and Nason (1995) in the hump shaped responses of reduced-form vector autoregressions to temporary impulses. One possible explanation for this behaviour is that a positive impulse to an economy in the neighbourhood of point B may so improve the quality of financial intermediation that the entire equilibrium switches temporarily from the lower to the upper branch of the equilibrium correspondence. It is not yet clear *why* the economy would revert automatically to the lower branch without an offsetting negative stimulus. The precise response pattern may depend critically on how external shocks influence the process by which the economy selects a branch from its equilibrium correspondence. In a companion paper (Azariadis and Chakraborty, 1998), we employ a class of selection mechanisms, which mix systematic and random factors, to study the excess volatility of capital asset prices.

UCLA

Appendix: The Optimum Loan Contract

An investor's payoff from a contract δ is

$$U = \begin{cases} 0 & \text{if } \theta \leqslant x, \\ b(\theta\rho - R^L) = b\rho(\theta - x) & \text{if } \theta > x. \end{cases}$$

Expected payoff of the investor is therefore

$$U(\delta) = b\rho[1 - \mu(x)] \tag{A.1}$$

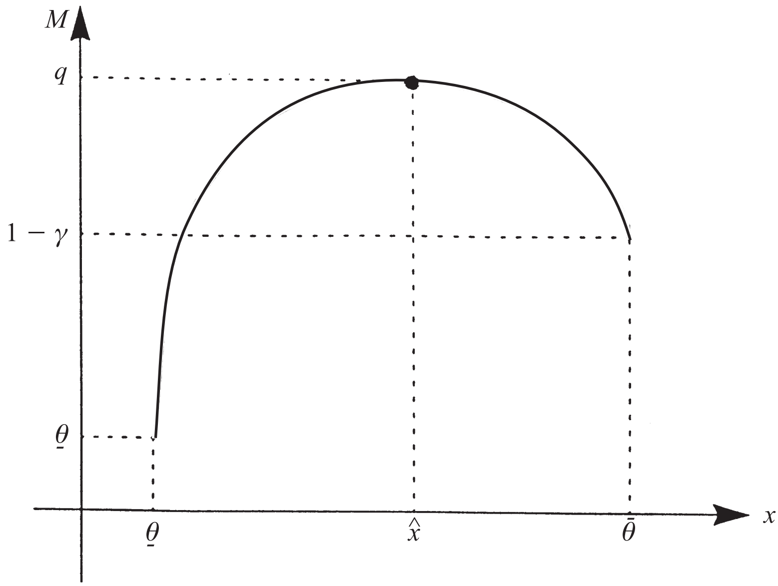
where $\mu(x) = \mathrm{E}\min(\theta,\ x) = x[1 - H(x)] + \int_{\underline{\theta}}^{x} \theta\, dH$. Similarly, a bank's payoff is

$$\pi^B = \begin{cases} b(\rho\theta - \rho\Gamma - R^D) & \text{if } \theta \leqslant x, \\ b(\rho x - R^D) & \text{if } \theta > x. \end{cases}$$

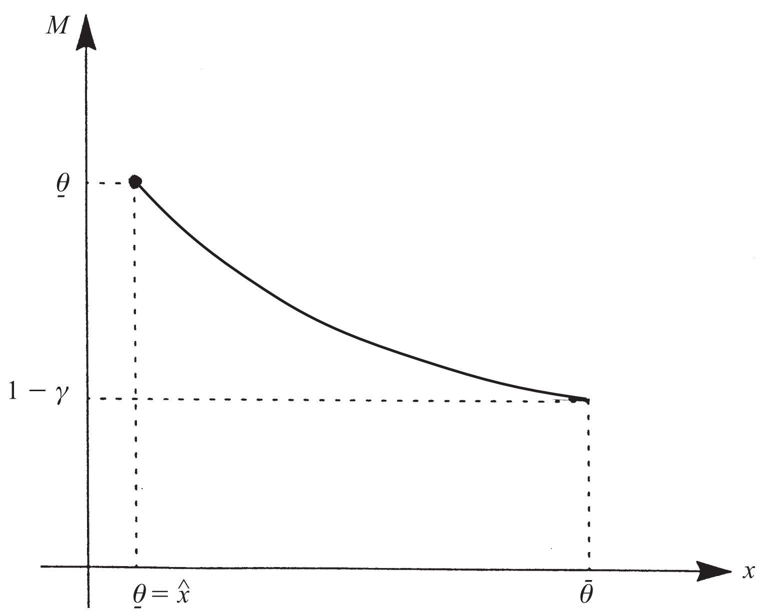
The verification cost is assumed to be proportional to the size of the loan, e.g., $\Gamma = \gamma b$, where $\gamma \in [0,\ \underline{\theta}]$.

Define $q \equiv R^D/\rho$ and let

$$M(x, \gamma) \equiv x[1 - H(x)] + \int_{\underline{\theta}}^{x} (\theta - \gamma)\, dH. \tag{A.2}$$



Case I: $\gamma L(\underline{\theta}) < 1$



Case II: $\gamma L(\underline{\theta}) > 1$

Fig. A.1. *The Optimum Loan Contract*

Expected bank profit is then,[9]

$$\Pi(\delta,\ q) \equiv \rho b[M(x,\gamma) - q]. \tag{A.3}$$

We assume free entry into banking, so that equilibrium bank profits must be zero.

DEFINITION 1. *Given* (q,γ), *the loan contract* $\hat{\delta} = (b,\ \hat{x},\ R^L)$ *is optimal if:*

(i) $R^L = \rho x$, *and*
(ii) $\hat{x} = \arg\max \Pi(\delta,\ q)$ *subject to* $U(\delta) \geqslant U_0$.

Suppose now that the likelihood function $\mathscr{L}(\theta) = h(\theta)/[1 - H(\theta)]$, defined for $\theta \in [\underline{\theta},\ \overline{\theta}]$, is increasing in θ. Then the critical state $\hat{x}$ defined above satisfies

$$\hat{x} = \arg\max_{x \in \Theta} M(x,\gamma). \tag{A.4}$$

Thus, $\hat{x}$ solves the equation

$$M_x(x,\gamma) = [1 - H(x)][1 - \gamma\mathscr{L}(x)] \leqslant 0, \tag{A.5}$$

with equality if $x > \underline{\theta}$. Since $\Gamma = \gamma b$, $M(\underline{\theta},\gamma) = \underline{\theta}$ and $M(\overline{\theta},\gamma) = 1 - \gamma$, Fig. A.1 shows that there exists a critical value of γ,

$$\gamma_c = \min\{1,\ 1/h(\underline{\theta})\},$$

and a weakly decreasing function of γ, $\hat{x}$: $[0,\ 1] \to \Theta$, such that

$$\hat{x}(0) = \overline{\theta},\ \hat{x}(\gamma) = \underline{\theta} \quad \forall \gamma \in [\gamma_c,\ 1]. \tag{A.6}$$

Furthermore, free entry into banking implies that $\Pi(\delta,\ q) = 0$, so that from (A.3), the exogenous price q must satisfy

$$q = \begin{cases} M(\hat{x},\gamma) & \text{if } \gamma \in [0,\ \gamma_c], \\ \underline{\theta} & \text{if } \gamma \in [\gamma_c,\ 1]. \end{cases} \tag{A.7}$$

Theorem 2 below describes completely the optimum contract.

THEOREM 2. *Given* γ *and* (A.6), *the optimum loan contract* $\hat{\delta} = (\hat{b},\ \hat{x},\ \hat{R}^L)$ *satisfies* (A.4) *and* $U = U_0$, i.e., $\hat{b} = U_0/\rho\{1 - \mu[\hat{x}(\gamma)]\}$. *In particular,* $\gamma \in (\gamma_c,\ 1]$ *implies no state verification and no interest rate spread* $(x = \underline{\theta},\ R^L = R^D)$, *while* $\gamma \in [0,\ \gamma_c)$ *implies verification in some states and a positive interest rate spread* $(x > \underline{\theta},\ R^L > R^D)$.

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[9] If the last term in (A.2) is negative, the loan contract is not renegotiation proof. The bank would throw away a bankrupt project rather than monitor it; it should then be possible to change $(b,\ x)$ and improve the contract by lowering x instead of throwing away θb worth of resources. To ensure that bankrupt projects are not abandoned we assume γ is 'small', i.e., $\gamma \in [0,\ \underline{\theta}]$ which guarantees $\int_{\underline{\theta}}^{x} (\theta - \gamma)\ dH \geqslant 0$.

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