

Reply to comment by P. Duval and M. Montagnat on “Superplastic deformation of ice: Experimental observations”

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1. Background

[1] In a recent paper, we reported the results of a detailed experimental investigation of the rheological properties of ice [Goldsby and Kohlstedt, 2001]. One of the innovative aspects of our investigation was the use of fine-grained samples that enabled us to explore deformation not only by grain-size insensitive mechanisms involving grain matrix dislocation processes but also by grain-size sensitive flow mechanisms involving grain boundary sliding (GBS). An exciting discovery in our research was an extensive creep regime in which GBS contributes substantially to the flow of ice. Most importantly, this deformation regime in which basal slip is accommodated by GBS, referred to in our paper as the “superplastic flow” regime, governs the flow of ice over a wide range of temperature, grain size and stress conditions found in glaciers, ice sheets and icy planetary interiors.

[2] In their comment on our paper, Duval and Montagnat [2002] repeat the objection that they had raised previously [Montagnat and Duval, 2000] concerning our conclusion that superplastic deformation controls the flow of glaciers and ice sheets. They assert “that GBS as a significant creep mechanism is not compatible with observations on the development of fabrics and microstructures in ice sheets. Special emphasis is placed [instead] on the role of grain boundary migration as a recovery process in glacier ice.” They then continue by arguing that basal slip accommodated by grain boundary migration (GBM) rather than by GBS is the dominant deformation mechanism in ice.

[3] In this reply, we extend the results presented in detail by Goldsby and Kohlstedt [1997] and Goldsby and Kohlstedt [2001] to further demonstrate that GBS is a very important deformation process in natural ice bodies. Our reasoning is based on three points: (1) The microstructures reported for samples from large natural ice bodies are remarkably similar to those observed in samples deformed

in the laboratory in the superplastic flow regime. (2) Our constitutive equation for ice is in excellent agreement with the flow behavior determined from field measurements on glaciers and ice sheets. (3) Contrary to the assertion of Montagnat and Duval [2000] and Duval and Montagnat [2001], fabric development is fully compatible with deformation in the superplastic regime described in our paper. In addition, we note that, while GBM associated with dynamic recrystallization is an important recovery mechanism in ice, it is not a deformation mechanism and hence cannot accommodate basal slip.

2. Comparison of Microstructural and Mechanical Data From Laboratory and Field Experiments

[4] Deformation-induced microstructures observed in naturally and experimentally deformed ice are characteristic of the active deformation mechanism. Straight grain boundaries, equant grains, and the presence of four-grain junctions indicate a significant contribution from GBS involving grain switching. Irregular, sutured grain boundaries, flattened grains, and the absence of four-grain junctions signal the importance of dislocation creep and dynamic recrystallization via GBM.

[5] In their comment on our paper, Duval and Montagnat [2001] claim that “GBS as a significant creep mechanism is not compatible with . . . microstructures in ice sheets.” This statement is not true. Microstructures in glaciers and ice sheets are remarkably similar to those produced in the deformation experiments of Goldsby and Kohlstedt [1997, 2001]. For example, microstructural observations of ice from the Barnes Ice Cap in Canada revealed straight grain boundaries and numerous four-grain junctions [Hooke, 1973]. In a line drawing of the grain boundary morphology from thin sections in micrograph C of Figure 3 of Hooke [1973], we count 22 four-grain junctions. In fact, numerous four-grain junctions and straight grain boundaries are pervasive in all of the micrographs in Hooke’s paper, much as in the images of our samples.

[6] In the same paper, Hooke [1973] determined a stress exponent of $n = 1.65$ from strain measurements in an ice tunnel within the Barnes Ice Cap combined with a stress

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analysis; this value is in excellent agreement with our value of $n = 1.8$ for the GBS-accommodated basal slip regime [Goldsby and Kohlstedt, 1997, 2001]. Furthermore, the absolute magnitude of the strain rate as a function of stress determined by Hooke [1973] is in excellent agreement (differing by a factor of <2) with extrapolations of our constitutive equation [Goldsby and Kohlstedt, 2001, equation (3)] using appropriate values of grain size and temperature, as shown in Figure 3b of Peltier *et al.* [2000].

[7] We conclude that the remarkable similarity between microstructures in the Barnes Ice Cap and those observed by Goldsby and Kohlstedt [1997] in their superplastic creep regime, combined with the excellent agreement between the mechanical data from the laboratory and the field, demonstrates that GBS-accommodated basal slip with $n = 1.8$ is the dominant deformation process at the conditions of Hooke's [1973] field experiment. In addition to the favorable comparison of our mechanical data with Hooke's field data, extrapolation of our data to appropriate grain size, temperature, and stress conditions is also in excellent agreement with mechanical data from a number of other field studies [see Peltier *et al.*, 2000]. We also note that recent field data of Cuffey *et al.* [2000a, 2000b] demonstrate that the flow of ice in the Meserve Glacier is sensitive to grain size, consistent with deformation via GBS-accommodated basal slip.

[8] Finally, we emphasize that microstructures revealed by detailed analyses of ice cores through the Antarctica and Greenland ice sheets [e.g., Gow and Williamson, 1976; Thorsteinsson *et al.*, 1997; Gow *et al.*, 1997] are also consistent with those expected for superplastic flow in the upper reaches of these ice sheets, followed by a transition to dislocation creep in the deepest parts of the ice sheets. Microstructural analyses of the GRIP ice core from Greenland [Thorsteinsson *et al.*, 1997] reveal a polygonal, equiaxed microstructure in the upper ~ 2800 m of the ice sheet. Numerous four-grain junctions are present in the ice from the GRIP core at 2806 m based on the upper photograph of a petrographic thin section in Figure 3 of De La Chapelle *et al.* [1998]. Duval and coworkers have interpreted this microstructure to indicate dynamic recrystallization via subgrain rotation [e.g., De La Chapelle *et al.*, 1998]. As in the case of the Barnes Ice Cap, we maintain that the straight grain boundaries, equiaxed grains and numerous four-grain junctions are indicative of GBS-accommodated basal slip. Below ~ 2800 m at the GRIP site, microstructures are more characteristic of recrystallization via GBM with more irregular, curved grain boundaries. (It should be noted, however, that the observed grain boundaries are not as strongly sutured as often observed for materials undergoing dynamic recrystallization via GBM.) The increasing importance of strain-induced GBM below ~ 2800 m occurs because the grain size, temperature, and stress conditions at the base of the ice sheet are such that the ice deforms very close to the transition from the superplastic creep regime to the dislocation creep regime.

[9] To illustrate this point, we calculate the strain rate at three representative depths within the Greenland ice sheet at the GRIP site using equation (3) of Goldsby and Kohlstedt [2001] with appropriate values of stress, grain size, and temperature; these values are plotted as solid symbols on the effective strain rate versus effective stress plot of Figure

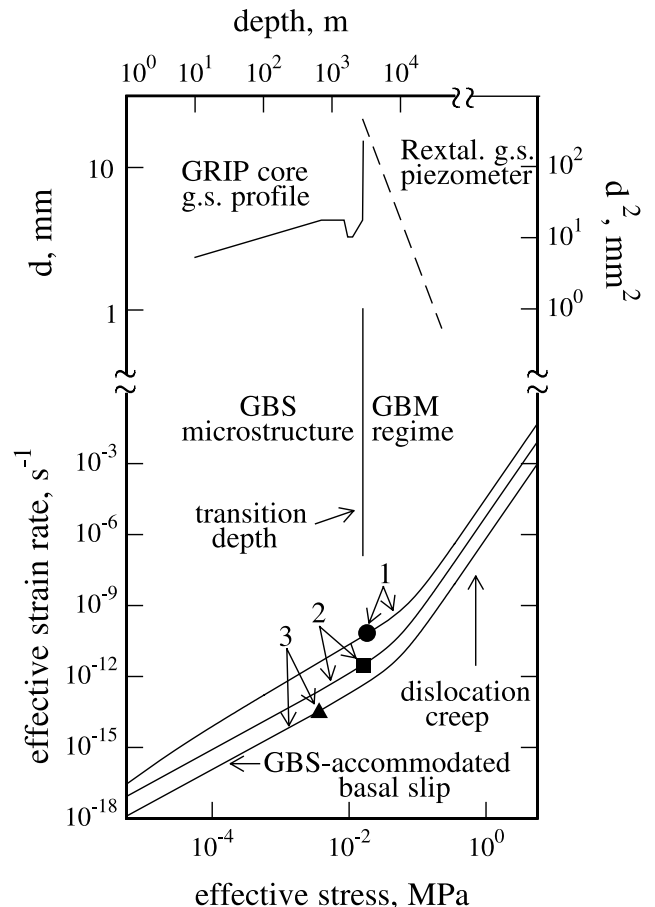


Figure 1. The top part is a plot of grain size versus depth on the upper axis and effective stress on the lower axis for the Greenland ice sheet. The grain size versus depth profile was determined from analyses of the GRIP core [De La Chapelle *et al.*, 1998]. The recrystallized grain-size piezometer is from Jacka and Jun [1994]. The bottom part is a plot of effective strain rate versus depth on the upper axis and effective stress on the lower axis. Solid circles are stress-strain rate data determined at discrete depths within the ice sheet. Continuous curves are calculated stress-strain rate values based on the constitutive equation, equation (3), of Goldsby and Kohlstedt [2001]. Data and curves are for three pairs of temperature and grain size: (1) $T = 273$ K, $d = 20$ mm; (2) $T = 258$ K, $d = 4$ mm; and (3) $T = 241$ K, $d = 4$ mm.

1. The shear stress τ is calculated via the relation $\tau = \rho g z \sin \alpha$, where ρ is the density, g is the acceleration due to gravity, z is the depth in the ice, and α is the surface slope. We use the grain size versus depth and temperature vs. depth data from De La Chapelle *et al.* [1998]. In addition to the strain rates determined at discrete points within the ice, we calculated the continuous strain rate vs. stress curves in Figure 1 from equation (3) of Goldsby and Kohlstedt [2001] for bounding values of grain size and temperature. As shown in Figure 1, deformation occurs within the superplastic (i.e., GBS-accommodated basal slip) creep regime throughout the ice sheet; at the greatest depths, however, the ice sheet deforms very close to the transition to the dislocation creep regime. The transition between the observed GBS and GBM microstructures also occurs at a

depth very close to the transition to the dislocation creep regime. The increasing contribution of strain-induced GBM to the observed microstructures therefore results from the increasing contribution of dislocation creep (with dislocation slip on both basal and nonbasal slip systems) to the strain rate. Hence the change in microstructure in the deepest portions of the ice sheets is consistent with GBS-accommodated basal slip with an increased contribution to the strain rate from dislocation creep.

[10] In addition to the changes in grain size, grain shape and grain boundary morphology discussed above, an abrupt change in lattice preferred orientation (LPO) occurs at a depth of ~ 2800 m in the Greenland ice sheet. A single maximum fabric that increases in strength with depth above 2800 m becomes an incomplete girdle fabric below 2800 m [e.g., *De La Chapelle et al.*, 1998]. This abrupt change in LPO occurs in close proximity to the transition to the dislocation creep regime, as shown in Figure 1, and is caused by dislocation slip on nonbasal as well as basal slip systems. Thus we suggest that the single maximum fabrics in the upper part of the ice sheet are indicative of GBS-accommodated basal slip and that the emergence of a single girdle fabric in the deepest parts of the ice represents increasing contributions to strain from dislocation creep.

[11] In the context of the present discussion, it should be emphasized that the $n = 3$ creep regime referred to by *Montagnat and Duval* [2000] is an artifact, as pointed out by *Goldsby and Kohlstedt* [2001]. A flow law with a stress exponent of $n = 3$, often referred to as the Glen flow law, arises because data used in identifying this regime were collected at the transition between the dislocation creep regime with $n = 4.0$ and the superplastic creep regime with $n = 1.8$. This point is further illustrated by *Durham et al.* [2001], who obtained creep results essentially identical to those reported in our paper.

3. Fabric Development During Superplastic Deformation

[12] *Duval and Montagnat* [2001] used the viscoplastic self-consistent model outlined by *Tomé* [1999] to calculate the fabric for ice, which they compare to fabrics in ice samples from the GRIP ice core. Unfortunately, they do not specify the parameters used in their calculation. What was the state of stress applied in their simulation? Was basal slip the only dislocation process considered? If not, what were the strengths of the pyramidal and prismatic slip systems relative to the basal slip system? Without this information, it is impossible to evaluate the validity of their conclusion that, based on differences between the observed and calculated fabrics, “GBS as a dominant deformation mode is clearly not compatible with the development of fabrics in ice sheets.” We emphasize that it is essential to consider more than just basal slip plus GBS in models of fabric development in ice. On the basis of the results of *Goldsby and Kohlstedt* [1997, 2001] and *Durham et al.* [2001], even in the regime that we referred to as superplastic flow, glide and climb on nonbasal slip systems contribute to deformation. This point is nicely illustrated in Figures 1, 2, 3, and 7 and quantified in equation (3) of *Goldsby and Kohlstedt* [2001]. The assumption made by *Duval and Montagnat* [2002] that only basal slip and GBS

operate in the superplastic creep regime is overly restrictive and not consistent with experimental observations.

[13] When GBS and dislocation slip are mutually accommodating mechanisms, as in the GBS-accommodated basal slip regime (i.e., the $n = 1.8$, $m = 1.4$ regime of *Goldsby and Kohlstedt* [2001], where m is the grain-size exponent) in ice, and a significant amount of strain is provided by dislocation slip, the development of LPO is expected [*Etheridge and Wilkie*, 1979]. It is important to reemphasize that in the $n = 1.8$ creep regime for ice, not all of the strain is due to GBS. In fact, creep in the $n = 1.8$ creep regime is dominated by dislocation slip on the basal slip system but rate-limited by GBS. Hence significant amounts of strain are provided by dislocation slip on the basal slip system (and to a lesser extent nonbasal slip systems). The absence of grain flattening after large strains [see *Goldsby and Kohlstedt*, 1997] indicates that GBS contributes substantially to the strain in the $n = 1.8$ creep regime; however, this lack of flattening cannot be taken to indicate that a significant part of the strain is not provided by basal slip, since GBM (which must be active during superplastic flow) acts to help maintain an equiaxed microstructure.

[14] The claim that “[GBS] is obviously not compatible with the development of fabrics” [*Duval and Montagnat*, 2001] is disproved by numerous studies in rock physics as well as materials science. An instructive example of texture development during superplastic flow in the geophysical literature is deformation of fine-grained calcite rocks [*Schmid et al.*, 1987; *Rutter et al.*, 1994]. In these studies, a LPO develops with increasing strain in both the dislocation creep regime and in a superplastic flow regime (characterized by $n = 1.7$ and $m = 1.9$) in which dislocation slip and GBS are mutually accommodating. The maxima in the orientation of poles to specific crystallographic planes developed in this superplastic creep regime are similar to those produced in the dislocation creep regime. *Schmid et al.* [1987, p. 783] pointed out that “some features of the texture found in the intracrystalline slip [dislocation creep] regime are maintained in the grain boundary sliding regime. This strongly suggests that a minor fraction of the total strain [in the superplastic regime] is still taken up by intracrystalline slip.” Similar experimental observations led *Rutter et al.* [1994, p. 1445] to conclude that “the presence of a CPO [crystallographic preferred orientation, synonymous with LPO] in intensely deformed, fine-grained calcite rocks cannot be considered diagnostic only of intracrystalline plastic flow [dislocation creep]... to the exclusion of grain boundary sliding.”

[15] Several observations from the metallurgical literature also demonstrate that LPO development is consistent with superplastic flow (K. Padmanabhan, personal communication, 2001). Textural studies in superplastic materials often focus on the effect of an initial texture formed by hot working, for example, on superplastic flow properties, in an effort to optimize shaping of materials with commercial applications as well as to understand the physics of superplastic flow. Mechanical treatments such as hot working, used to establish a fine grain size in a starting material, induce a strong initial LPO. Superplastic deformation of a material with an initial texture can preserve and in some cases strengthen certain texture peaks inherited from the starting material. For example, in the

Al phase of the Al-6 wt% Cu-0.3 wt% Zr alloy, $\{110\}\langle 1\bar{1}2\rangle$ texture orientations were stabilized by dislocation slip during superplastic flow [Bricknell and Edington, 1979]. In the two-phase Sn-38 wt% Pb alloy, the intensity of some texture components in both phases increased as a result of dislocation slip during superplastic deformation [Melton *et al.*, 1975]. During superplastic deformation of two-phase Zn-22 wt% Al and Zn-40 wt% Al alloys, one of the three basal plane texture peaks present in the starting material was retained and one of these basal plane maxima was strengthened; these textural changes were attributed to slip on $\{10\bar{1}1\}\langle \bar{1}2\bar{1}0\rangle$ and $\{11\bar{2}2\}\langle \bar{1}\bar{1}23\rangle$ slip systems [Melton and Edington, 1973].

[16] In other cases, new texture components form as the result of superplastic deformation. In the eutectic alloy Zn-5 wt% Al, a new pole figure maximum formed as a result of basal slip during superplastic deformation [Edington *et al.*, 1976]. In the Al phase of the two-phase alloy Zn-22 wt% Al, new slip-stabilized LPOs were introduced during superplastic deformation [Melton and Edington, 1974]. As these examples demonstrate, the maintenance, strengthening and, in some cases, formation of new texture peaks are fully consistent with superplastic deformation and are generally attributed to the dislocation slip that accommodates grain boundary sliding in this regime [e.g., Edington *et al.*, 1976].

4. Role of Recrystallization During Dislocation Creep

[17] Dynamic recrystallization is an important recovery mechanism during dislocation creep. As grain boundaries migrate through regions of high dislocation density, they leave behind dislocation-free material. In a region in which dislocation density is increasing, the flow stress also increases, that is, the material work hardens. Steady state flow cannot be attained unless dislocations are removed by dislocation-dislocation annihilation, formation and progressive rotation of subgrain boundaries, or grain boundary migration. This point is the basis for the paper by Montagnat and Duval [2000] in which a steady state flow law is developed by balancing dislocation production against dislocation removal by GBM.

[18] The point not recognized by Montagnat and Duval [2000] and Duval and Montagnat [2002] is that GBM is not a deformation mechanism, consequently, it does not produce strain [McLean, p. 307, 1967]. As a result, in their model, Montagnat and Duval are left with a single deformation mechanism, namely, basal slip, to account for all of the creep strain of ice. As discussed by von Mises [1928], five independent slip systems are required for a polycrystalline material to flow homogeneously without opening up voids [see also Paterson, 1969]. This condition can be relaxed somewhat if deformation is inhomogeneous, in which case, four independent slip systems are adequate [Hutchinson, 1976]. If dislocation climb operates in addition to glide, then three slip independent slip systems will suffice [Groves and Kelly, 1969]. In addition, grain boundary and grain matrix diffusion as well as grain boundary sliding will also allow grains to deform to an arbitrary shape. Therefore Duval and Montagnat [2001] have incorrectly asserted that "grain boundary migration ... can accommodate basal slip. The flow law with a stress expo-

nent of about 2 could be related to the efficiency of GBM in relieving the internal stress field resulting from high plastic anisotropy of the ice crystal." In fact, as demonstrated here, this flow law is appropriately described as basal slip accommodated by GBS, that is, the superplastic flow regime with $n = 1.8$ of Goldsby and Kohlstedt [1997, 2001].

5. Summary

1. For ice, a flow regime in which basal slip is accommodated by GBS dominates over a wide range of conditions that overlap those found in glaciers, ice sheets, and planetary interiors.

2. The use of fine-grained ice samples permits grain-size sensitive creep experiments to be performed at steady state under laboratory conditions.

3. The constitutive equation for flow of ice reported by Goldsby and Kohlstedt [2001] based on convincing laboratory results provides an excellent description of deformation in natural ice bodies.

4. The development of lattice preferred orientation is fully compatible with deformation in the superplastic (i.e., basal slip accommodated by GBS) creep regime due to dislocation slip on basal and, to a lesser extent, on nonbasal slip systems in this regime.

5. While an important recovery process, grain boundary migration is not a strain-producing deformation mechanism. Hence any model built simply on basal slip accommodated by grain boundary migration [e.g., Montagnat and Duval, 2000] is invalid because it does not fulfill even a relaxed von Mises criterion, which is necessary for flow of a polycrystalline material without opening of voids.

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