MATH 251–252: ARITHMETIC EXPECTED WITHOUT CALCULATORS

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Calculators, of any kind, are prohibited on all quizzes, exams, etc. in Math 251. There are apparently calculators available with the capability of connecting to the internet, and ones with computer algebra systems capable of some symbolic differentiation and integration. Obviously such calculators can't be allowed, but I have no way to tell whether any particular calculator has these capabilities. Therefore the only thing I can do is a blanket prohibition.

This means that you are expected to be able to do some arithmetic without a calculator. For example, you can't rely on a calculator to evaluate things like the following:

$$17 + 33, \qquad 2 \cdot 9.007, \qquad \sin(0), \qquad \sqrt{49}, \qquad \text{and} \qquad 0.02^2.$$

This file is about the sorts of arithmetic that it is expected you can do without a calculator, and some things you will not be expected to do (except possibly in extra credit problems).

1. Arithmetic expected without calculators

Here is a (not necessarily complete) list of arithmetic you might be expected to do.

(1) Any two digit addition, such as $34+78$ or $34+(-78)$. Also arithmetic which is similarly easy because of zeroes: $34{,}000+78$, $34{,}000+78{,}000$, $0.0034 + 0.0078$, $17.0034 + 0.0078$, etc.

(2) Any single digit multiplication (up to $9 \cdot 9$, multiplying anything by 2, 3, or any power of 10, or combinations (such as $20 \cdot 311 = 6220$), and anything which is similarly easy because of zeroes, such as $0.03 \cdot 0.007 = 0.00021$ or $0.02^2 = 0.0004$.

(3) Division by 2, 3, 4, and 5 when it comes out evenly, and division by 10, 100, etc. even if it doesn't come out evenly. (Do not replace $\frac{33}{10}$ with 3.3 when exact answers are specified, which is most of the time. If the problem statement doesn't contain a decimal point, then almost certainly no decimal points should appear anywhere in the solution or final answer.)

(4) Square roots of perfect squares:

$$\sqrt{0} = 0, \qquad \sqrt{1} = 1, \qquad \sqrt{4} = 2, \qquad \sqrt{9} = 9, \qquad \ldots, \qquad \sqrt{100} = 10,$$

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as well as cases which are similarly easy because of zeroes, such as

$$\sqrt{900} = 30, \qquad \sqrt{10{,}000} = 100, \qquad \sqrt{250{,}000} = 500, \qquad \sqrt{0.04} = 0.2,$$

etc.

(5) A few small cube roots and higher, including $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[4]{16} = 2$, and anything similarly easy because of zeroes.

(6) Expressions like

$$\frac{1}{0.1} = 10, \qquad \frac{1}{0.0001} = 10{,}000, \qquad \frac{1}{0.02} = 50,$$

etc.

(7) Values of elementary functions at special points, such as

$$\sin(0) = 0, \qquad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \qquad \sin\left(\frac{\pi}{2}\right) = 1, \qquad \cos(3\pi) = -1,$$

$$e^0 = 1, \qquad \ln(1) = 0, \qquad \ln(e) = 1, \qquad \arctan(1) = \frac{\pi}{4},$$

and many similar things.

(8) I don't expect decimal approximations to $\sqrt{2}$ or π (see below), but you are expected to know

$$1 < \sqrt{2} < \sqrt{3} < 2 < \sqrt{5} < \cdots \qquad \text{and} \qquad 2 < e < 3 < \pi < 4.$$

2. Making easy arithmetic hard

Here are examples of calculations that might appear which are easy if done the right way, but become unsuitable without a calculator if done the wrong way.

Example 2.1. Suppose a mosquito takes off from a log and buzzes above a pool of stagnant water. Its height $h(t)$ above the surface of the water, in millimeters, at time t seconds, is modelled by $h(t) = 279 + 10t^2 - t^3$. You are asked for the average upwards velocity between 1 and 3 seconds.

This is

$$\begin{aligned}\frac{h(3) - h(1)}{3 - 1} &= \frac{279 + 10 \cdot 3^2 - 3^3 - (279 + 10 \cdot 1^2 - 1^3)}{2} \\ &= \frac{279 + 90 - 27 - (279 + 10 - 1)}{2} \\ &= \frac{\cancel{279} + 90 - 27 - \cancel{279} - 10 + 1}{2} = 27.\end{aligned}$$

You make the arithmetic much harder if you evaluate

$$h(3) = 279 + 10 \cdot 3^2 - 3^3 = 342$$

rather than cancelling 279 as above.

Example 2.2. You are given (or have calculated)

$$g'(x) = 3x^2 - 24x = 3x(x - 8),$$

and want to know the sign of $g'(9)$. It is easy to see that

$$g'(9) = 3 \cdot 9 \cdot (9 - 8) = 27 > 0.$$

Don't use

$$g'(x) = 3 \cdot 9^2 - 24 \cdot 9.$$

In fact, even to find the sign of $g'(35)$, you can write

$$g'(9) = 3 \cdot 35 \cdot (35 - 8) = 3 \cdot 35 \cdot 27,$$

which is obviously positive **without** actually multiplying out $3 \cdot 35 \cdot 27$.

Example 2.3. You are given

$$g(x) = x^3 - 12x^2 + 16,$$

and you are asked to find the values of x at which there is a local minimum. You find $x = 8$. The problem as given **does not ask for** the value of the function at $x = 8$. That is, you **don't** need to calculate $g(8)$. Read the problem to see what is actually asked for!

3. Arithmetic not expected

I will not expect on exams numerical calculations of the following types. (However, as above, I might well expect calculations which might look like these if done without paying attention. Also, such things might appear in extra credit problems.)

(1) If $f(x) = x^3 - 6x^2 + 11x - 9$, I won't expect calculations of $f(7)$ or $f(-9)$. (But I might expect a calculation of $f(10)$. Also, some calculations I do expect might turn out like this if done the wrong way. See Section 2.)

(2) I don't expect you to know decimal approximations to $\frac{1}{3}$, $\frac{1}{7}$, $\sqrt{2}$, $\sqrt{3}$, π, or e. In fact, most questions will ask for exact answers, and such decimal approximations, not being exact, will be considered wrong. If the answer is $2\sqrt[3]{70}$, just leave it like that. (But see (1), (8), (4), and (5) in Section 1.)