MATH 251 (PHILLIPS) QUIZ 5, 2 June 2025. 20 minutes; 20 points.

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Standard exam instructions apply. In particular, no calculators, no communication devices, and no notes except as 3×5 file card, written on both sides. Also, all notation must be correct, with "=", "lim", etc. everywhere they are supposed to be, and nowhere they are not supposed to be. Write answers on this page. Use the back if necessary. Total: 20 points.

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1. (10 points) Find the equation of tangent line to the graph of $f(x) = 3x^2 - x$ at $x = -2$. You need not calculate the derivative directly from the definition.

Solution. We need the slope, which is $f'(-2)$, and a point on the line, such as $(-2, f(-2)) = (-2, 3(-2)^2 - (-2)) = (-2, 14)$. Now $f'(x) = 6x - 1$, so $f'(-2) = -13$. Therefore the equation is $y - 14 = -13(x - (-2))$, which can be rearranged to give $y = -13x - 12$.

We want the slope at the *particular* value $x = -2$. Therefore we must substitute $x = -2$ in the formula for the derivative $f'(x)$ *before* using it as the slope of a line. The equation

$$\cancel{y - 4 = (6x - 1)(x - (-2))}$$

is wrong—it is not even the equation of a line. □

2. (10 points) Let $f(x) = -x^3 - 6x^2 - 5x + 9$. Find the intervals of concavity and the inflection points of f.

Solution. First,

$$f'(x) = -3x^2 - 12x \qquad \text{and} \qquad f''(x) = -6x - 12 = -6(x + 2).$$

So $f''(x) = 0$ when $x = -2$. Therefore there is a possible inflection point at $x = -2$.

On the interval $(-\infty, -2)$, we have $f''(x) > 0$. This is clear from the formula: $x + 2 < 0$, so $-6(x + 2) > 0$ (Since f'' is continuous, we can check at one point, say $x = -3$, which gives $f''(-3) = -6(-3 + 2) = 6 > 0$.)

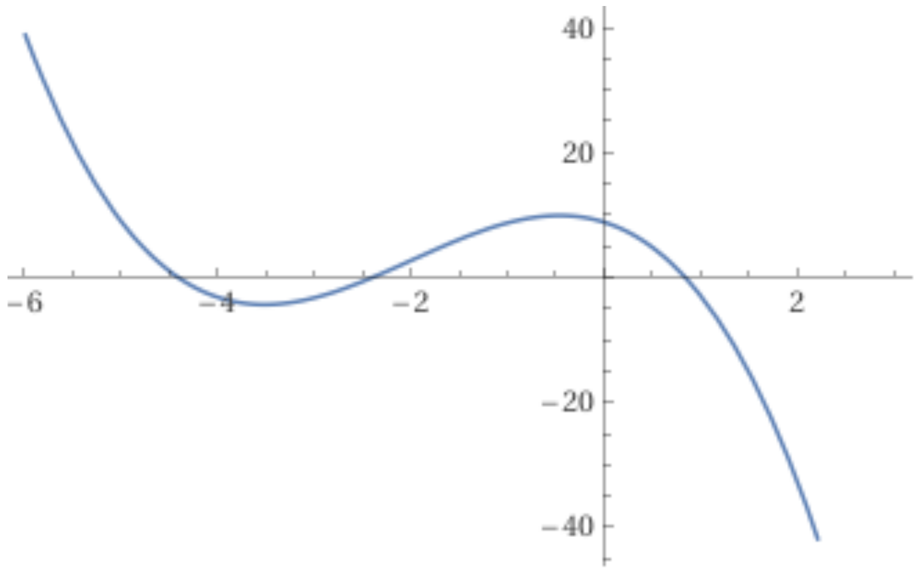
On the interval $(-2, \infty)$, we have $f''(x) < 0$. This is clear from the formula: $x + 2 > 0$, so $-6(x + 2) < 0$ (Since f'' is continuous, we can check at one point, say $x = 0$, which gives $f''(-3) = -6 \cdot 0 - 12 = -12 < 0$.)

Therefore:

- f is concave up on $(-\infty, -2)$.
- f is concave down on $(-2, \infty)$.
- f has an inflection point at $x = -2$, because concavity changes from up to down.

□

Here is a graph of the function (**not** required as part of the solution; given so you can compare your results with the actual graph) (on back of page):


40
20
−20
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2