WORKSHEET: DERIVATIVES AS RATES OF CHANGE

Names and student IDs: _____

See Section 3.4 of the book.

Recall: the derivative of something is its instantaneous rate of change. For example, if you are going 70 miles per hour along a long straight road, it means that, if you kept going at the same speed for an hour, you would be 70 miles farther along. If $P(x)$ is the profit in dollars Wang's Widgets Inc. makes when manufacturing x widgets, then saying $P'(300{,}000) = 6$ (in dollars per widget) means that, if the profit per additional widget is the same when the production level is between 300,000 and 300,001, then producing 300,001 widgets instead of 300,000 will generate an additional \$6 in profit. (This is really the linear approximation.)

1. A particle moves along a line. Time is measured in seconds, and position is measured in meters to the right of the origin. When $0 \leq t \leq 20$, the position is given by $p(t) = t^3 - 5t^2 - 2t$.

a. Where did the particle start?

b. What are the units of $p'(t)$?

c. What was the velocity of the particle at $t = 2$ (including units)? Was it moving left or right?

d. What are the units of $p''(t)$ (acceleration)?

e. At time 5 seconds, was the particle speeding up or slowing down? (Be careful.)

2. Let $P(t)$ be the population of Megalopolis, in millions of people, at time t is years, counted conventionally.

a. What are the units of $P'(t)$?

b. Suppose $P'(t) < 0$ when $2010 \leq t \leq 2020$. What does that say about the population?

3. Let $C(x)$ be the cost, in dollars, to Wang's Widgets Inc. of producing x widgets.

a. What are the units of $C'(x)$?

b. What does $C'(500{,}000)$ mean? (By the way, $C'(x)$ is called the *marginal cost.*)

c. Do you expect $C'(500{,}000)$ to be positive or negative? Why?

4. Let $T(x)$ be the temperature, in degrees Fahrenheit (°F), along Interstate 5, x miles north of Eugene at 11:00 am on 25 July 2024.

a. What are the units of $T'(x)$?

b. Do you think is is more likely that $T'(50) > 0$ or $T'(50) < 0$? Why?

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