WORKSHEET: INFLECTION POINTS; TANGENT LINES 1

Names and student IDs: _____

Recall:

- If $f''(x) > 0$ on an interval (a, b), then f is concave up on (a, b).
- If $f''(x) < 0$ on an interval (a, b), then f is concave down on (a, b).

1. Let $f(x) = x^4$, so $f''(x) = 12x^2$, and $f''(x) = 0$ only when $x = 0$. In the following steps, you will decide whether f has an inflection point at $x = 0$.

(a) What is the sign of $f''(x)$ on $(0, \infty)$?

(b) Is f concave up or concave down on $(0, \infty)$?

(c) What is the sign of $f''(x)$ on $(-\infty, 0)$?

(d) Is f concave up or concave down on $(-\infty, 0)$?

(e) Does f have an inflection point at $x = 0$?

2. Let $f(x) = x^4 - 6x^3$. Find the intervals of concavity and the inflection points. Start by finding $f''(x)$. Check where $f''(x)$ is zero, where $f''(x)$ is strictly positive, and where $f''(x)$ is strictly negative.

3. A fire-breathing monster was asked to find the equation of the tangent line to the graph of $g(x) = x^2 + 2x$ at $x = 2$. Its answer was $y = \cancel{(2x + 2)(x - 2)} + 8$.

Even without knowing the formula for g, this answer is *obviously* wrong. Why is it obviously wrong?

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