MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 8 PART 1

This homework sheet is due in class on Wednesday 21 May 2025 (week 8), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Total 18 points.

1. (18 points) Let $q(x) = -\frac{x^3}{3} + x^2 + 15x + 6$. Identify the open intervals on which q is increasing, those on which q is decreasing, and all critical points, local minimums, and local maximums.

Solution. We have

$$q'(x) = -x^2 + 2x + 15 = -(x^2 - 2x - 15) = -(x - 5)(x + 3).$$

The solutions to $q'(x) = 0$ are therefore $x = 5$ and $x = -3$. These are the critical points.

For x in the interval $(-\infty, -3)$, we have $x < -3$, so $x - 5 < 0$ and $x + 3 < 0$, whence $-(x - 5)(x + 3) < 0$. Therefore q is decreasing on $(-\infty, -3)$.

For x in the interval $(-3, 5)$, we have $x > -3$ and $x < 5$, so $x-5 < 0$ and $x+3 > 0$, whence $-(x-5)(x+3) > 0$. Therefore q is increasing on $(-3, 5)$.

For x in the interval $(5, \infty)$, we have $x > 5$, so $x-5 > 0$ and $x+3 > 0$, whence $-(x-5)(x+3) < 0$. Therefore q is decreasing on $(5, \infty)$.

Since q is continuous, q is decreasing on $(-\infty, -3)$, and q is increasing on $(-3, 5)$, it follows that q has a local minimum at -3.

Since q is continuous, q is increasing on $(-3, 5)$, and q is decreasing on $(5, \infty)$, it follows that q has a local maximum at 5. □

Since q' is continuous, the sign of q' on each of the intervals $(-\infty, -3)$, $(-3, 5)$, and $(5, \infty)$ can also be determined using test points. I chose -4, 0, and 6. At $x = -4$ and $x = 6$, the calculations are much easier using the formula $q'(x) = -(x - 5)(x + 3)$.

We have

$$q'(-4) = -(-4 - 5)(-4 + 3) = -9,$$

so $q'(x) < 0$ for x in $(-\infty, -3)$. Therefore q is decreasing on $(-\infty, -3)$.

We have $q'(0) = 15$ (using $q'(x) = -x^2 + 2x + 15$), so $q'(x) > 0$ for x in $(-3, 5)$. Therefore q is increasing on $(-3, 5)$.

We have

$$q'(6) = -(6 - 5)(6 + 3) = -9,$$

so $q'(x) < 0$ for x in $(5, \infty)$. Therefore q is decreasing on $(5, \infty)$.

Date: 21 May 2025.