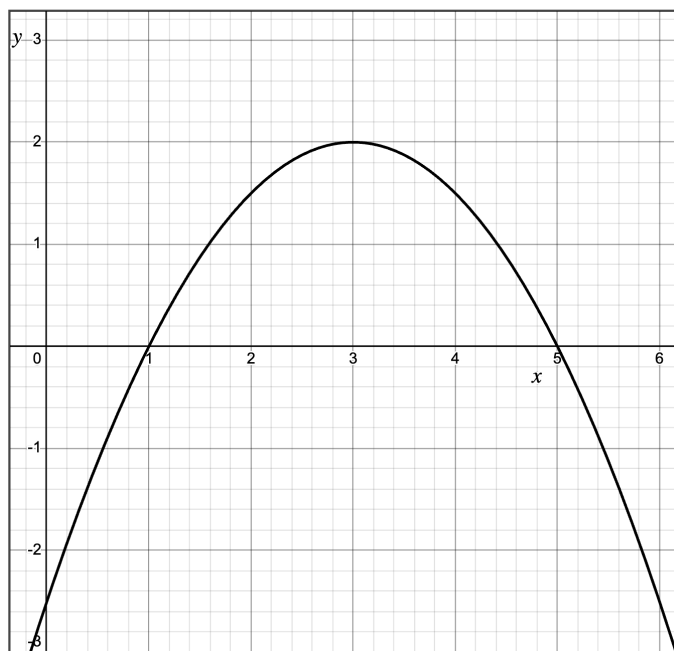


WORKSHEET: DERIVATIVES AND LOCAL EXTREMUMS

Names and student IDs: _____

1. Shown below is the graph of $y = h(x)$ for some differentiable function h .



Recall that $h'(c)$ is the slope of the tangent line to the graph of $y = h(x)$ at $x = c$ (the point $(c, h(c))$ on the graph).

Estimate $h'(1), h'(2), \dots, h'(5)$ (values of the derivative of h). (If you need to, draw tangent lines on the graph and estimate their slopes.)

$$h'(1) \approx \underline{\hspace{1cm}}, \quad h'(2) \approx \underline{\hspace{1cm}}, \quad h'(3) \approx \underline{\hspace{1cm}}, \quad h'(4) \approx \underline{\hspace{1cm}}, \quad h'(5) \approx \underline{\hspace{1cm}}.$$

2. Using the values you found, draw a graph of $y = h'(x)$, the **derivative** of $h(x)$.

3. Is the **derivative** $h'(x)$ of $h(x)$ is increasing on the interval $(0, 6)$, decreasing on $(0, 6)$, or increasing on parts of this interval and decreasing on other parts?

(Continued on back.)

4. Recall that a differentiable function f is increasing on an open interval (a, b) exactly when $f'(x) \geq 0$ on (a, b) , and that f is decreasing on (a, b) exactly when $f'(x) \leq 0$ on (a, b) .

Apply this with $f = h'$, and decide whether the **second derivative** $h''(x)$ of $h(x)$ is positive on the interval $(0, 6)$, negative on $(0, 6)$, or positive on parts of this interval and negative on other parts.

5. Draw the graph of some function $y = k(x)$ on the interval $(0, 6)$ such that $h'(3) = 0$ and $h''(x) > 0$ on all of $(0, 6)$. Does your function have a local minimum or a local maximum at $x = 3$? Is this function concave up or concave down on $(0, 6)$?

6. Let $g(x) = x^3 - 6x^2$. Then

$$g'(x) = 3x^2 - 12x = 3x(x - 4) \quad \text{and} \quad g''(x) = 6x - 12 = 6(x - 2).$$

By considering the signs of $g'(x)$ and $g''(x)$, find the following:

Critical points of g :

On which open intervals is g increasing?

On which open intervals is g decreasing?

Local maximums of g :

Local minimums of g :

On which open intervals is g concave up?

On which open intervals is g concave down?