MATH 251 (PHILLIPS) QUIZ 4, 19 May 2025. 20 minutes; 20 points.

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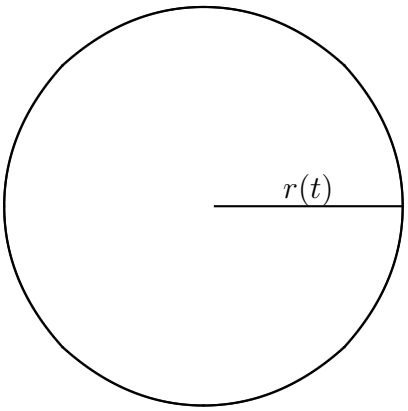
Standard exam instructions apply. In particular, no calculators, no communication devices, and no notes except as 3×5 file card, written on both sides. Also, all notation must be correct, with "=", "lim", etc. everywhere they are supposed to be, and nowhere they are not supposed to be. Write answers on this page. Use the back if necessary.

(20 points) A circular puddle of water on a hot sidewalk in Needles CA is evaporating. At 2:00 pm, it had a radius of 10 inches, and its radius was decreasing at 3 inches per hour. At that time, was its area increasing or decreasing? At what rate? (Be sure to include the correct units in your answer.)

Solution. Here is the picture. (The circle is not as round as it should be.)



As shown in the picture, let $r(t)$ be the radius of the puddle at time t, measured in inches and with time measured in hours past noon. Let $A(t)$ be the area at time t, measured in square inches. Both the area and the radius vary with time, so must be treated as functions, not constants.

The time we are interested in is $t = 2$. We are given $r(2) = 10$ and $r'(2) = -3$. (It is negative, since the radius is *decreasing*.) We want to know $A'(2)$. The functions $A(t)$ and $r(t)$ are related by the equation

$$A(t) = \pi\big[r(t)\big]^2.$$

(You are expected to know this formula.) Differentiate with respect to t:

$$A'(t) = \pi \cdot 2r(t)r'(t) = 2\pi r(t)r'(t).$$

(Don't forget the factor $r'(t)$! That will spoil the whole thing!) Evaluate this at $t = 2$, using $r(2) = 10$ and $r'(2) = -3$. This gives

$$A'(2) = 2\pi r(2)r'(2) = 2\pi(10)(-3) = -60\pi.$$

So the area is decreasing at the rate of 60π square inches per hour. (Don't forget the units!)

Here, for reference, is what the solution looks like in physicists' notation.

$$A = \pi r^2.$$

Differentiate with respect to t:

$$\frac{dA}{dt} = \pi \cdot 2r\frac{dr}{dt} = 2\pi r\frac{dr}{dt}.$$

(Don't forget the factor $\frac{dr}{dt}$!) Now substitute $r = 10$ and $\frac{dA}{dt} = -3$, getting $\frac{dA}{dt} = 2\pi(10)(-3) = -60\pi$. □