MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 7 PART 1.

This homework sheet is due in class on Wednesday 14 May 2025 (week 7), in class. Write answers on a separate piece of paper, well organized and well labelled, with **each solution starting on the left margin of the page**.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

For each of the following problems, do the following.

(1) Draw a picture of the situation.
(2) Name all quantities you will need to use in the solution. Include the units with the names, as in the example below. Label items in the picture with the names of the appropriate variables.
(3) Determine all the relations between the quantities in (1). Write the these relations as equations.

Do not attempt any further steps towards solving the problem.

See the example for Written Homework 5.

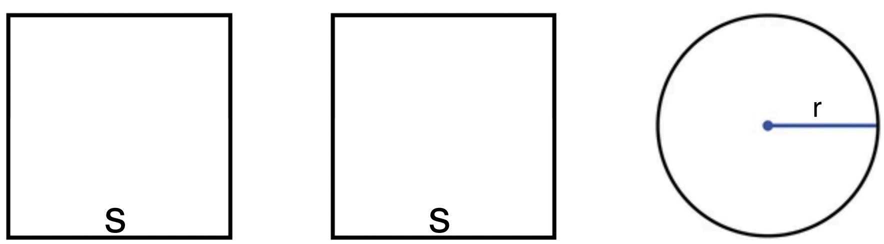
10 points per problem, total 30 points.

Warning: Solutions have not been properly proofread. Remember that there is extra credit for reporting errors!

Usually several solutions are possible.

1. A 31 inch long wire will be cut in three pieces. Two of the pieces will be the same length, and will be to form the perimeters of two squares, of course of the same size. The other piece will be used to form the perimeter of a circle. How should the wire be cut to minimize the combined area of the three shapes?

Solution. Here is a picture, with the appropriate quantities on it labelled:



As shown, r is the radius of the circle, and s is the length of one of the sides of the squares, both in inches. Also let C be the area of the circle, and let S be the area of one of the squares, both in square inches. Finally let A be the total area of all three figures. The perimeter of the circle is $2\pi r$ and the perimeter of one square is $4s$, so $2\pi r + 2 \cdot 4s = 31$, that is, $2\pi r + 8s = 31$. We know that $C = \pi r^2$ and $S = s^2$, and we have $A = C + 2S$. $\square$

It is helpful, but not necessary, to name the separate areas. Without doing this, one gets the relations

$$2\pi r + 8s = 31 \qquad \text{and} \qquad A = \pi r^2 + 8s^2.$$

Alternate solution (sketch). Let p be the perimeter of one of the squares, and let c be the circumference of the circle. Also let A be the total area. (These are not labelled in the picture.) The squares then have side length $\frac{p}{4}$, so areas $\left(\frac{p}{4}\right)^2$. The circle has radius $\frac{c}{2\pi}$, so has area $\pi\left(\frac{c}{2\pi}\right)^2 = \frac{c^2}{4\pi}$. Thus:

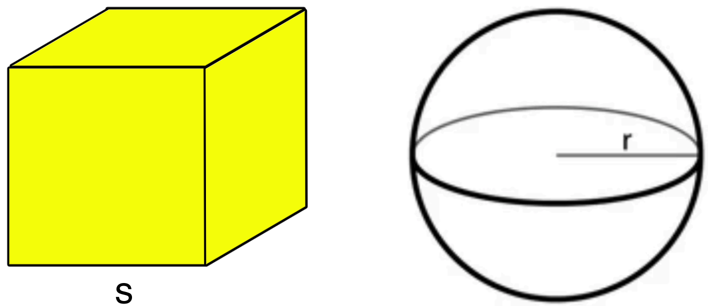
$$c + 2p = 31 \qquad \text{and} \qquad A = 2\left(\frac{p}{4}\right)^2 + \frac{c^2}{4\pi} = \frac{p^2}{8} + \frac{c^2}{4\pi}.$$

$\square$

Date: 14 May 2025.

2. A thin film is to be used to form the surfaces of a sphere and a cube. If you are supposed to use exactly 11 square feet of this film, how should the film be divided to minimize the combined volume of the two objects? Assume there is no waste.

Solution. Here is a picture, with the appropriate quantities on it labelled:



We let r be the radius of the sphere, and let s be the length of one of the sides of the cube, both in feet. Also let C be the volume of the cube, and let S be the volume of the sphere, both in cubic feet. Finally let V be the total volume of both objects. The surface area of the cube is $6s^2$ and the surface area of the sphere is $4\pi r^2$, so $4\pi r^2 + 6s^2 = 11$. We know that $C = \frac{4}{3}\pi r^3$ and $S = s^3$, and we have $V = C + S$. □

It is helpful, but not necessary, to name the separate volumes. Without doing this, one gets the relations

$$4\pi r^2 + 6s^2 = 11 \qquad \text{and} \qquad V = \frac{4}{3}\pi r^3 + s^3.$$

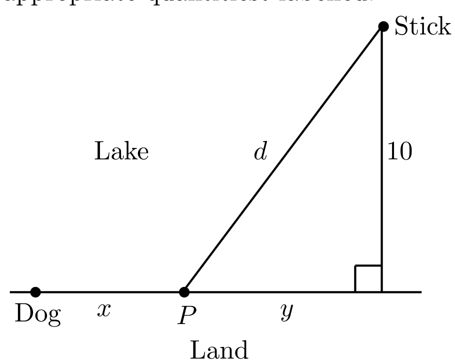
Alternate solution (sketch). Let C and S be as in the first solution, let A be the surface area of the sphere, and let B be the surface area of the cube. (These are not labelled in the picture.) One can figure out the relations between the volume and surface areas of spheres and cubes, getting

$$C = \left(\frac{B}{6}\right)^{3/2} \qquad \text{and} \qquad S = \frac{1}{6}\left(\frac{A}{\pi}\right)^{3/2}.$$

□

3. A dog is standing on a straight lakeshore. A robot throws a stick in the water, which lands 10 feet away from the shoreline, with the closest point on the shoreline being 13 feet away from the dog. The dog runs at 23 feet per second and swims at 3 feet per second. Find the fastest route the dog can take to the stick. (The dog will run along the shore to some point, and then swim diagonally to reach the stick.)

Solution. Here is a picture, with the appropriate quantitiest labelled:



Here P is the ;point at which the dog enters the water. As shown, x is the distance from the location of the dog to P, y is the distance from P to the point on the shoreline that is closest to the stick, and d is the distance the dog swims. Also let T be the total time the trip takes, in minutes. The relations are:

$$T = \frac{x}{23} + \frac{d}{3}, \qquad x + y = 13, \qquad \text{and} \qquad d^2 = y^2 + 10^2.$$

□