MATH 251 (PHILLIPS) QUIZ 3, Tuesday 13 May 2025. 20 minutes; 20 points.

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Standard exam instructions apply. In particular, no calculators, no communication devices, and no notes except a 3×5 file card, written on both sides. Also, all notation must be correct, with "=", "lim", etc. everywhere they are supposed to be, and nowhere they are not supposed to be. Write answers on this page. Use the back if necessary.

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1. (20 points) If $\cos(y) = (4x - y)^3 - \arctan(17)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

Solution. Let's write it with y as an explicit function $y(x)$ of x:

$$\cos(y(x)) = (4x - y(x))^3 - \arctan(17).$$

Differentiate both sides with respect to x, using the chain rule on both sides:

$$-\sin(y(x))y'(x) = 3(4x - y(x))^2 \frac{d}{dx}(4x - y(x)) = 3(4x - y(x))^2 \left(4 - y'(x)\right).$$

(The derivative of arctan(17) is zero because arctan(17) is a constant.)

Now solve for $y'(x)$:

$$\begin{aligned} -\sin(y(x))y'(x) &= 12(4x - y(x))^2 - 3(4x - y(x))^2 y'(x) \\ 3(4x - y(x))^2 y'(x) - \sin(y(x))y'(x) &= 12(4x - y(x))^2 \\ y'(x) &= \frac{12(4x - y(x))^2}{3(4x - y(x))^2 - \sin(y(x))}. \end{aligned}$$

This expression can't be further simplified.

For those who prefer the other notation, here it is written with $\dfrac{dy}{dx}$. Differentiate with respect to x, using the chain rule on both sides, just as before:

$$-\sin(y)\frac{dy}{dx} = 3(4x - y)^2 \frac{d}{dx}(4x - y) = 3(4x - y)^2 \left(4 - \frac{dy}{dx}\right).$$

(The derivative of arctan(17) is zero because arctan(17) is a constant.)

Now solve for $\dfrac{dy}{dx}$:

$$\begin{aligned} -\sin(y)\frac{dy}{dx} &= 12(4x - y)^2 - 3(4x - y)^2 \frac{dy}{dx} \\ 3(4x - y)^2 \frac{dy}{dx} - \sin(y)\frac{dy}{dx} &= 12(4x - y)^2 \\ \frac{dy}{dx} &= \frac{12(4x - y)^2}{3(4x - y)^2 - \sin(y)}. \end{aligned}$$

As before, this expression can't be further simplified. □