MATH 251 (PHILLIPS): SOLUTIONS TO WRITTEN HOMEWORK 6 PART 1

This homework sheet is due in class on Tuesday 6 May 2025 (week 6), in class. It is review for some of the algebra needed for implicit differentiation. Therefore, if possible, please do it before Monday's class. The method is the same in all five problems.

Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print this page and write on it, using the back for the second problem if needed.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated, total 20 points.

1. (5 points.) Solve the following equation for D:

$$2D - 3 = 5D + 8.$$

Check your solution to be sure it is correct.

Solution. To make the method completely clear, more steps are shown than is required.

$$\begin{aligned} 2D - 3 &= 5D + 8 \\ 2D - 5D &= 8 + 3 \\ -3D &= 11 \\ D &= -\frac{11}{3}. \end{aligned}$$

Checking the solution:

$$2\left(-\frac{11}{3}\right) - 3 = -\frac{22}{3} - \frac{9}{3} = -\frac{31}{3} \qquad \text{and} \qquad 5\left(-\frac{11}{3}\right) + 8 = -\frac{55}{3} + \frac{24}{3} = -\frac{31}{3}.$$

So $D = -\dfrac{11}{3}$ really does solve the equation. □

2. (5 points.) Solve the following equation for D:

$$10 - 3(2D + 5) = 11 - 8D.$$

To be sure your solution is correct, check it by substituting it back into the equation.

Solution. To make the method completely clear, more steps are shown than is required.

$$\begin{aligned} 10 - 3(2D + 5) &= 11 - 8D. \\ 10 - 6D - 15 &= 11 - 8D. \\ -6D - 5 &= -8D + 11 \\ -6D + 8D &= 11 + 5 \\ 2D &= 16 \\ D &= \frac{16}{2} = 8. \end{aligned}$$

Checking the solution:

$$10 - 3(2 \cdot 8 + 5) = 10 - 48 - 15 = -53 \qquad \text{and} \qquad 11 - 8 \cdot 8 = 11 - 64 = -53.$$

So $D = 8$ really does solve the equation. □

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3. (5 points.) Solve the following equation for D:

$$7 + x(r + sD) = b + cD.$$

Solution. To make the method completely clear, more steps are shown than is required.

$$\begin{gathered} 7 + x(r + sD) = b + cD \\ 7 + xr + xsD = b + cD \\ xsD - cD = b - 7 - xr \\ (xs - c)D = b - 7 - xr \\ D = \frac{b - 7 - xr}{xs - c}. \end{gathered}$$

The last expression can't be usefully further simplified. (Be sure you understand why!) □

4. (5 points.) Solve the following equation for D:

$$1 - \sin(xy)(y + xD) = 6x + 12y^2 D.$$

Solution. To make the method completely clear, more steps are shown than is required.

$$\begin{gathered} 1 - \sin(xy)(y + xD) = 6x + 12y^2 D \\ 1 - \sin(xy) \cdot y - \sin(xy) \cdot xD = 6x + 12y^2 D \\ -\sin(xy) \cdot xD - 12y^2 D = 6x - 1 + \sin(xy) \cdot y \\ \big(-x\sin(xy) - 12y^2\big)D = 6x - 1 + y\sin(xy) \\ D = \frac{6x - 1 + y\sin(xy)}{-x\sin(xy) - 12y^2} = -\frac{6x - 1 + y\sin(xy)}{x\sin(xy) + 12y^2}. \end{gathered}$$

The last expression can't be usefully further simplified. (Be sure you understand why!) □

This problem came from implicit differentiation of $x + \cos(xy) = 3x^2 + 4y^3$.