WORKSHEET SOLUTIONS: IMPLICIT DIFFERENTIATION 2

Names and student IDs: <u>Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]</u>

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Further reminders: in implicit differentiation problems, y (or some other variable) is implicitly a function of x (or some other variable). So, for example, $\dfrac{d}{dx}(y^3) = 3y^2\dfrac{dy}{dx}$, not zero (and certainly not ~~$3y^2$~~—that is **never** right).

Also, $\dfrac{dy}{dx}(x^2y + y^6)$ means the product of $\dfrac{dy}{dx}$ and $x^2y + y^6$. It does **not** mean the derivative of $x^2y + y^6$ with respect to x. That is correctly written $\dfrac{d}{dx}(x^2y + y^6)$. Getting this wrong is a serious error.

You will use implicit differentiation to find $\dfrac{dy}{dx}$ when $y^7 = \tan(3x - y) + \pi^3$. You **must** solve for $\dfrac{dy}{dx}$.

1. Rewrite the formula with y written as a function of x.

Solution. $y(x)^7 = \tan(3x - y(x)) + \pi^3$. $\square$

2. There are two places you will need the chain rule. What are they?

Solution. When differentiating y^7 (or $y(x)^7$), and when differentiating $\tan(3x - y)$ (or $\tan(3x - y(x))$). $\square$

3. Carry out the implicit differentiation. (You will solve for $\dfrac{dy}{dx}$ in Problem 4.)

Solution written using $y'(x)$. We have:

$$y(x)^7 = \tan(3x - y(x)) + \pi^3.$$

Differentiate both sides with respect to x, using the chain rule on both sides:

$$7[y(x)]^6y'(x) = \sec^2(3x - y(x))\frac{d}{dx}(3x - y(x)) = \sec^2(3x - y(x))\left(3 - y'(x)\right).$$

(The derivative of π^3 is zero because π^3 is a constant.) $\square$

Solution written using $\dfrac{dy}{dx}$. Differentiate with respect to x, using the chain rule on both sides, just as before:

$$7y^6\frac{dy}{dx} = \sec^2(3x - y)\frac{d}{dx}(3x - y) = \sec^2(3x - y)\left(3 - \frac{dy}{dx}\right).$$

(The derivative of π^3 is zero because π^3 is a constant.) $\square$

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4. In the result of Problem 3, solve for $y'(x)$ or $\frac{dy}{dx}$ (depending on which notation you used).

Solution written using $y'(x)$. We have

$$7[y(x)]^6 y'(x) = \sec^2(3x - y(x))\left(3 - y'(x)\right).$$

Solve for $y'(x)$:

$$\begin{aligned} 7[y(x)]^6 y'(x) &= 3\sec^2(3x - y(x)) - \sec^2(3x - y(x))y'(x) \\ 7[y(x)]^6 y'(x) + \sec^2(3x - y(x))y'(x) &= 3\sec^2(3x - y(x)) \\ y'(x) &= \frac{3\sec^2(3x - y(x))}{7[y(x)]^6 + \sec^2(3x - y(x))}. \end{aligned}$$

This expression can't be further simplified. □

Solution written using $\frac{dy}{dx}$. We have

$$7y^6 \frac{dy}{dx} = \sec^2(3x - y)\left(3 - \frac{dy}{dx}\right).$$

Solve for $\frac{dy}{dx}$:

$$\begin{aligned} 7y^6 \frac{dy}{dx} &= 3\sec^2(3x - y) - \sec^2(3x - y)\frac{dy}{dx} \\ 7y^6 \frac{dy}{dx} + \sec^2(3x - y)\frac{dy}{dx} &= 3\sec^2(3x - y) \\ \frac{dy}{dx} &= \frac{3\sec^2(3x - y)}{7y^6 + \sec^2(3x - y)}. \end{aligned}$$

As before, this expression can't be further simplified. □