WORKSHEET SOLUTIONS: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if $h(x) = f(g(x))$ for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Also,

$$\frac{d}{dx}\big(e^x\big) = e^x \qquad \text{and} \qquad \frac{d}{dx}\big(\ln(x)\big) = \frac{1}{x}.$$

For each of the following functions, differentiate the function and simplify the derivative, or else tell me that no differentiation rule you have seen so far applies:

1. $f(x) = x\ln(x) - x$.

Solution. Use the product rule: $f'(x) = \dfrac{d}{dx}\big(x\big)\ln(x) + x\ln'(x) - 1 = \ln(x) + x\big(\tfrac{1}{x}\big) - 1 = \ln(x)$. □

2. $g(x) = e^{x^2+7x}$.

Solution. Use the chain rule. Write $e^y = \exp(y)$ for clarity. (This is standard notation.) So $g(x) = \exp(x^2 + 7x)$ and

$$g'(x) = \exp'(x^2+7x)\frac{d}{dx}\big(x^2+7x\big) = \exp(x^2+7x)(2x+7) = (2x+7)e^{x^2+7x}.$$

Note: ~~$\exp(x^2 + 7x)2x + 7$~~ is wrong, because of missing parentheses. □

3. $q(x) = \ln(x^2 + e^x)$.

Solution. Use the chain rule:

$$q'(x) = \ln'(x^2+e^x)\frac{d}{dx}\big(x^2+e^x\big) = \left(\frac{1}{x^2+e^x}\right)(2x+e^x) = \frac{2x+e^x}{x^2+e^x}.$$

This expression can't be further simplified. (See the Midterm Zero problems.) □

4. $s(x) = e^{x^2\sin(x)}$.

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Solution. Use the chain rule and the product rule. Write $e^y = \exp(y)$. Then

$$\begin{aligned} s'(x) &= \exp'(x^2\sin(x))\frac{d}{dx}\big(x^2\sin(x)\big) \\ &= \exp(x^2\sin(x))\big(2x\sin(x)+x^2\cos(x)\big) = e^{x^2\sin(x)}\big(2x\sin(x)+x^2\cos(x)\big). \end{aligned}$$

□

5. $p(x) = 7^x$.

Solution. No rule we have seen applies. (Not the power rule: that is for functions like x^7, in which the exponent is a constant.)

It can, however, be done. Write $7 = e^{\ln(7)}$, so

$$7^x = \big(e^{\ln(7)}\big)^x = e^{\ln(7)x} = \exp\big(\ln(7)x\big).$$

Now use the chain rule:

$$p'(x) = \exp'\big(\ln(7)x\big)\frac{d}{dx}\big(\ln(7)x\big) = \exp\big(\ln(7)x\big)\cdot\ln(7) = \ln(7)\cdot 7^x.$$

□

6. $a(x) = x^{\sin(x)}$.

Solution. No rule we have seen applies. (Not the power rule: that is for functions like x^7, in which the exponent is a constant. Not the rule for e^x either: that is for functions like $e^{\sin(x)}$, in which the base is a constant.)

It can, however, be done. Write $x = e^{\ln(x)}$, so

$$x^{\sin(x)} = \big(e^{\ln(x)}\big)^{\sin(x)} = e^{\ln(x)\sin(x)} = \exp\big(\ln(x)\sin(x)\big).$$

Now use the chain rule, and the product rule on the insides:

$$\begin{aligned} p'(x) &= \exp'\big(\ln(x)\sin(x)\big)\frac{d}{dx}\big(\ln(x)\sin(x)\big) \\ &= \exp\big(\ln(x)\sin(x)\big)\big(\ln'(x)\sin(x)+\ln(x)\sin'(x)\big) \\ &= \exp\big(\ln(x)\sin(x)\big)\left(\frac{\sin(x)}{x}+\ln(x)\cos(x)\right). \end{aligned}$$

□