MATH 251 (PHILLIPS): WRITTEN HOMEWORK 4 PART 2

This homework sheet is due in class on Wednesday 23 April 2025 (week 4), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page. Or, print this page and write on it, using the back for the second problem if needed.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

One problem: 20 points.

1. (20 points.) James Tutt Snodgrass III has finished Math 251 and gone home for the holidays to his mother's farm, only to be asked to solve the following problem:

His mother wants to fence off a rectangular enclosure for llamas. The west side of the enclosure will be a wall which is already present. The north side will be an old fence which will cost \$10 per foot to adequately reinforce. The remaining two sides will consist of new fencing costing \$20 per foot. What are the dimensions of the largest enclosure that can be made for \$6000?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

Recall the list of steps on page 441 of the book, as further subdivided in class:

(1) (a) Draw a picture if possible. (Most solution files don't include pictures.)
    (b) Name all the quantities which appear in the problem.
(2) (a) What quantity is to be minimized or maximized, and which of these is it?
    (b) What are the restrictions on the values of the variables? (Include "degenerate" cases if possible.)
(3) Express the quantity to be minimized or maximized (from (2a)) in terms of (some of) the other variables.
(4) (a) Relate the variables in the formula in (2a) to each other.
    (b) Use the relations in (4a) to write the quantity to be minimized or maximized as a function of a single variable.
(5) Determine the domain of the function in (4b) from the physical problem to be solved. (For example, lengths of fences can't be negative.)
(6) Use methods of calculus to find the absolute minimum or maximum of the function in (4b), as appropriate, on the interval in (5).

In (6), it is **not** sufficient to simply find the critical points; you must check that your function has an absolute minimum or maximum on the interval in (5) at the value of the variable you choose. (Don't use the second derivative test, which has not yet been discussed. This problem is designed to be accessible via the methods we have already seen.)

Date: 17 April 2025.