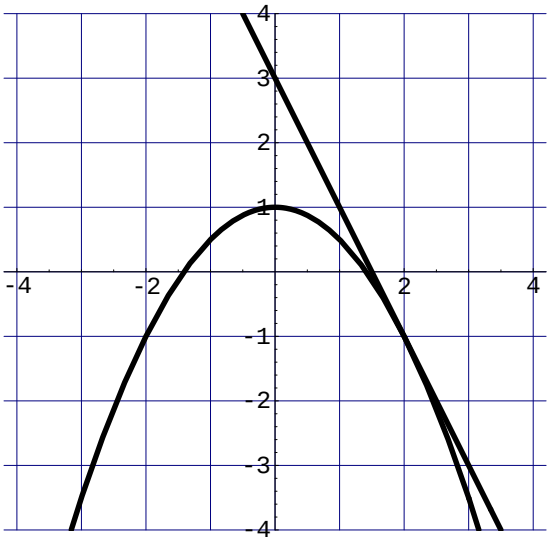
MATH 251 (PHILLIPS): WRITTEN HOMEWORK 4 PART 1

This homework sheet is due in class on Tuesday 22 April 2025 (week 4), in class. Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated, total 46 points.

1. (10 points.) The picture below shows the graph of a function $y = f(x)$ and the tangent line to the graph at $x = 2$.



Let g be the function $g(x) = \dfrac{f(x)}{x^2+7}$. Find $g'(2)$. **Do not try to guess a formula for $f(x)$; use only the information provided.**

2. (10 points.) Let f and g be functions which are differentiable at -2 and which satisfy

$$f(-2) = -5, \qquad f'(-2) = -3, \qquad g(-2) = 4, \qquad \text{and} \qquad g'(-2) = 2.$$

Let $w(x) = x - f(x)g(x)$ for all x. Find $w'(-2)$.

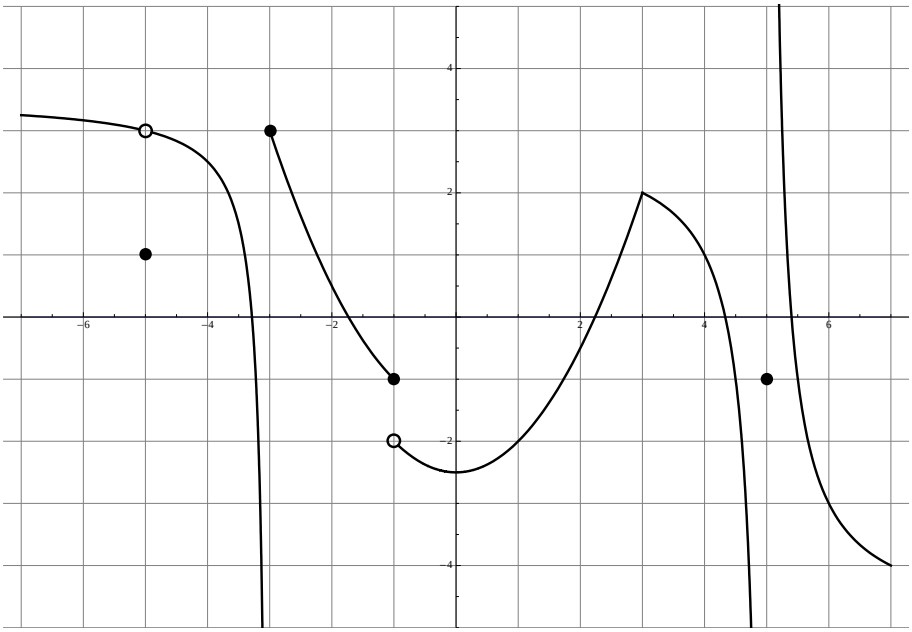
3. (10 points.) Use the methods of calculus to find the exact values of x at which the function $f(x) = \left(3x^2 - 5x - 5\right) e^{-x-7}$ takes its absolute minimum and maximum values on the interval $[2, 10]$.

Hint: $f'(x) = \left(-3x^2 + 11x\right) e^{-x-7}$.

Also, you will probably want a calculator to evaluate f at the critical points etc.

4. (2 points/part) For the function $y = g(x)$ graphed below, answer the following questions:

Date: 17 April 2025.



(To make this faster to do and grade, the problems do **not** ask for reasons, but you should be able to give reasons. Reasons will be included in the solutions.)

(a) List all points a in $(-7, 7)$ at which g is neither continuous nor differentiable.

(b) List all points a in $(-7, 7)$ at which g is continuous but not differentiable.

(c) List all points a in $(-7, 7)$ at which g is differentiable but not continuous.

(d) List all points a in $(-7, 7)$ for which $\lim_{x \to a} g(x)$ does not exist.

(e) List all points a in $(-7, 7)$ for which $\lim_{x \to a} g(x)$ does exist but is not equal to $g(a)$. For each of these points, find $\lim_{x \to a} g(x)$.

(f) List all points a in $(-7, 7)$ such that $g'(a) = 0$.

(g) Give a rough estimate of $g'(1)$, or state that $g'(1)$ does not exist.

(h) Give a rough estimate of $g'(4)$, or state that $g'(4)$ does not exist.