

MATH 251 (PHILLIPS) QUIZ 1, Monday 21 April 2025. 20 minutes; 20 points.

NAME: SOLUTIONS

Student id: $\pi\pi\pi-\pi\pi-\pi\pi\pi\pi$

Standard exam instructions apply. In particular, no calculators, no communication devices, and no notes except as 3×5 file card, written on both sides. Also, all notation must be correct, with “=”, “lim”, etc. everywhere they are supposed to be, and nowhere they are not supposed to be. Write answers on this page. Use the back if necessary.

1. (7 points) Differentiate the function $h(x) = \cos(3x^2 - 16x)$. (You need not do this directly from the definition.)

Solution: Use the chain rule:

$$\begin{aligned} h'(x) &= \cos'(3x^2 - 16x) \cdot \frac{d}{dx}(3x^2 - 16x) \\ &= -\sin(3x^2 - 16x) \cdot (3 \cdot 2x - 16) = -(6x - 16) \sin(3x^2 - 16x). \end{aligned}$$

Parentheses are required in the first and second steps: if

$$\frac{d}{dx} 3x^2 - 16x, \quad -\sin(3x^2 - 16x) \cdot 3 \cdot 2x - 16,$$

or anything similar appears in your work, it is wrong.

2. (7 points) Differentiate the function $w(t) = \frac{\sin(t)}{3t^4 + t + 6}$. (You need not do this directly from the definition.)

Solution: Use the quotient rule:

$$\begin{aligned} w'(t) &= \frac{\sin'(t) \cdot (3t^4 + t + 6) - \sin(t) \frac{d}{dt}(3t^4 + t + 6)}{(3t^4 + t + 6)^2} \\ &= \frac{\cos(t)(3t^4 + t + 6) - \sin(t)(12t^3 + 1)}{(3t^4 + t + 6)^2} \\ &= \frac{(3t^4 + t + 6) \cos(t) - (12t^3 + 1) \sin(t)}{(3t^4 + t + 6)^2}. \end{aligned}$$

The last step is conventional, but not required.

Parentheses are required around $3t^4 + t + 6$ and $12t^3 + 1$. If they are missing, the solution is wrong.

3. (6 points) Find the exact value of $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 + x - 12}$, or explain why it does not exist. (This problem is not primarily about notation, but **notation counts**.)

Solution: This limit has the indeterminate form “ $\frac{0}{0}$ ”, so work is needed. We factor the denominator, and then cancel common factors in the fraction:

$$\lim_{x \rightarrow 3} \frac{x - 3}{x^2 + x - 12} = \lim_{x \rightarrow 3} \frac{x - 3}{(x - 3)(x + 4)} = \lim_{x \rightarrow 3} \frac{1}{x + 4} = \frac{1}{3 + 4} = \frac{1}{7}.$$