MATH 251 (PHILLIPS): WRITTEN HOMEWORK 3

This homework sheet is due **in class** on Wednesday 16 April 2025 (week 3). Write answers on a separate piece of 8.5 by 11 inch paper, well organized and well labelled, with each solution starting on the left margin of the page.

All the requirements in the sheet on general instructions for homework apply. In particular, show your work (unlike WeBWorK), give exact answers (not decimal approximations), and **use correct notation.** (See the course web pages on notation.) Some of the grade will be based on correctness of notation in the work shown.

Point values as indicated; total 50 points.

1. (10 points.) This problem is mostly about using correct notation. Accordingly, most of the credit is for correctness of notation. See the discussion in class Wednesday 9 April.

Consider the problem of finding the derivative of the function $m(t) = 2t^7 - bt^{-2} - \pi^3$, in which b is a *constant*.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and differentiation symbols everywhere they belong, and nowhere else. Start with "$m'(t)$". Use $\dfrac{d}{dx}$ to indicate differentiation with respect to x, $\dfrac{d}{dt}$ to indicate differentiation with respect to t, etc., with appropriate parentheses.

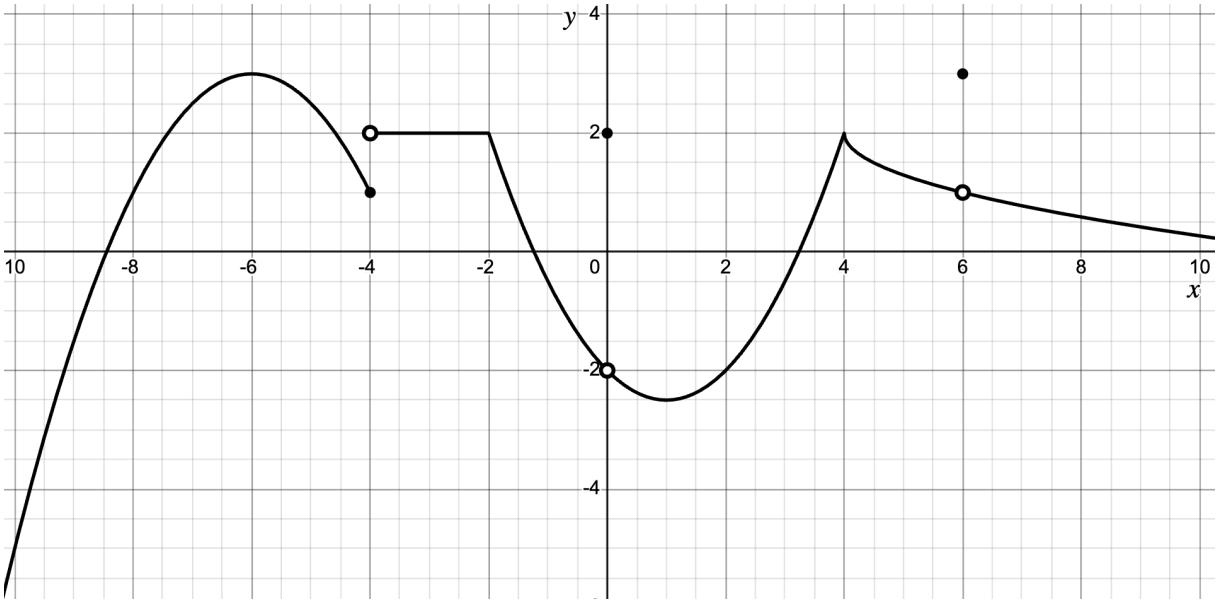
Show at least the following steps:

(1) Using the sum, difference, and constant multiple rules for derivatives.
(2) Using the power rule on each of the steps from (1), including the "-1" part.
(3) Simplification of the expression resulting from (2).

For textbook discussion of the next problem, see the graphs on the following pages, and the associated discussion: 135, 140, 143, 145, 151, 180, 182, 235–237, and 239. (We will return to infinite limits.)

It is very important that you understand limits, continuity, and derivatives in terms of the pictures. Otherwise, the pictures I draw on the board during class etc. will not make sense.

2. (5 points/part) For the function $y = w(x)$ graphed below, answer the following questions.



(a) Does $\lim_{x \to -4} w(x)$ exist? If so, what is it? If not, why not?

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(b) Does $\lim_{x\to 6} w(x)$ exist? If so, what is it? If not, why not?

(c) What is the largest interval containing 2 on which w is continuous? Why?

(d) Which of the following best describes $w'(-9)$? Why?
(1) $w'(-9)$ does not exist.
(2) $w'(-9)$ is close to 0.
(3) $w'(-9)$ is positive and not close to 0.
(4) $w'(-9)$ is negative and not close to 0.
(5) None of the above.

(e) Which of the following best describes $w'(-6)$? Why?
(1) $w'(-6)$ does not exist.
(2) $w'(-6)$ is close to 0.
(3) $w'(-6)$ is positive and not close to 0.
(4) $w'(-6)$ is negative and not close to 0.
(5) None of the above.

(f) List all points in $(-10, 10)$ at which w is not differentiable. Give reasons.

3. (10 points.) Find the derivative of the function $R(t) = 4at^3 - t^2\cos(t) - \pi^2$. ($a$ is a *constant*.)

4. (10 points.) Find the derivative of the function $h(s) = \sqrt{12 + s^2 + \sin(s)}$.