

## WORKSHEET SOLUTIONS: FINDING THE ABSOLUTE MINIMUM AND MAXIMUM

Names and student IDs: Solutions  $[\pi\pi\pi-\pi\pi-\pi\pi\pi\pi]$

Recall the method for maximizing and minimizing a continuous function  $f$  on a closed interval  $[a, b]$ :

- (1) Find all the critical numbers  $c$  of  $f$  in  $[a, b]$ , that is, numbers  $c$  in  $[a, b]$  such that  $f'(c) = 0$  or  $f'(c)$  does not exist.
- (2) Evaluate  $f$  (not  $f'$ !) at all the numbers  $c$  found in (1) and at the endpoints  $a$  and  $b$ .
- (3) The absolute maximum of  $f$  on  $[a, b]$  is the largest value you got in (2). It occurs at the value of  $x$  you used to produce it. (In rare cases, the absolute maximum can occur at more than one value of  $x$ .) Similarly, the absolute minimum of  $f$  on  $[a, b]$  is the smallest value you got in (2), etc.

Let  $h(x) = e^{x/2} (164 - 73x + 7x^2)$ . Use the methods of calculus to find the exact values of  $x$  (not calculator approximations) at which  $h$  has its maximum and minimum values on the interval  $[-1, 5]$ .

You don't yet know how to find  $h'(x)$ . So here it is:  $h'(x) = \frac{1}{2} (7x - 3)(x - 6)e^{x/2}$ .

Also, the problem does not ask you to find the exact maximum and minimum values of  $h$  on the interval, only the exact values of  $x$  at which they occur.

1. Find all critical numbers of  $h$ . (Since  $h'$  is already factored, this is easy.)

*Solution.* Since  $h$  is differentiable everywhere, we only need to solve  $h'(c) = 0$ , that is,

$$\frac{1}{2} (7c - 3)(c - 6)e^{c/2} = 0.$$

Since  $e^{c/2}$  is never zero, there are exactly two solutions, namely  $c = 6$  and  $c = \frac{3}{7}$ . □

2. Which critical numbers are in  $[-1, 5]$ ?

*Solution.* Only  $\frac{3}{7}$  is in  $[-1, 5]$ . □

3. List the values of  $x$  at which one should evaluate  $h$  according to Step (2) at the top of the page.

*Solution.* Use  $x = -1$ ,  $x = 5$  (the endpoints), and  $x = \frac{3}{7}$  (the critical number in  $[-1, 5]$ ). □

4. Presumably using a calculator, evaluate  $h$  at each of the numbers just found. (Do **not** evaluate  $h'$  at those numbers! But **do** explain why it is not helpful to do that.)

*Solution.* I got

$$h(-1) \approx 147.993, \quad h\left(\frac{3}{7}\right) \approx 166.023, \quad \text{and} \quad h(5) \approx -316.745.$$

We need the values of  $h$  because we are trying to decide at which values of  $x$  the function  $h$  has its largest and smallest values.

In any case, we already know that  $h'\left(\frac{3}{7}\right) = 0$ . After all, that is how we found the number  $\frac{3}{7}$ . □

5. Where does  $h$  take its absolute minimum and maximum values on  $[-1, 5]$ ?

*Solution.* Since, from the previous calculation,  $h(5) < h(-1) < h\left(\frac{3}{7}\right)$ , the absolute maximum value occurs at  $x = \frac{3}{7}$  and the absolute minimum value occurs at  $x = 5$ . □

The value  $h(6) \approx -441.882$  is smaller than any of these, but we don't use it since 6 is not in the interval  $[-1, 5]$ .