Names and student IDs: _____

Chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if
$$h(x) = f(g(x))$$
for all x (in a suitable open interval), then
$$h'(x) = f'(g(x)) \cdot g'(x).$$

Example: If $h(x) = \sin(x^3)$ then write $h(x) = f(g(x))$ with
$$f(u) = \sin(u) \qquad \text{and} \qquad g(x) = x^3.$$

Thus
$$h'(x) = f'(g(x)) \cdot g'(x) = \sin'(x^3) \cdot \frac{d}{dx}(x^3) = \cos(x^3) \cdot 3x^2 = 3x^2 \cos(x^3).$$

Caution: "$\sin'(x^3)$" means you take the derivative of the sine function and evaluate it at x^3. It does *not* mean $\frac{d}{dx}(\sin(x^3))$. The expression $\frac{d}{dx}(\sin(x^3))$ means $h'(x)$, the derivative with respect to x of the function whose value at x is $\sin(x^3)$.

Now differentiate the following functions.

1. Let $w(x) = \cos(\sin(x))$. First, if we want to usefully write $w(x) = f(g(x))$, then
$$f(u) = \qquad\qquad\qquad \text{and} \qquad g(x) =$$
so
$$f'(u) = \qquad\qquad\qquad \text{and} \qquad g'(x) =$$
Now, if $w(x) = \cos(\sin(x))$ then $w'(x) =$

2. Let $p(x) = (2x^3 + 7x + 8)^{28}$. First, if we want to usefully write $p(x) = f(g(x))$, then
$$f(u) = \qquad\qquad\qquad \text{and} \qquad g(x) =$$
so
$$f'(u) = \qquad\qquad\qquad \text{and} \qquad g'(x) =$$
Now, $\frac{d}{dx}\left((2x^3 + 7x + 8)^{28}\right) =$

3. Let $q(x) = \sqrt{2x^3 + 7x + 8}$. First, if we want to usefully write $q(x) = f(g(x))$, then
$$f(u) = \qquad\qquad\qquad \text{and} \qquad g(x) =$$
so
$$f'(u) = \qquad\qquad\qquad \text{and} \qquad g'(x) =$$
Now, if $q(x) = \sqrt{2x^3 + 7x + 8}$ then $q'(x) =$

4. Let $m(x) = \cos(x\sin(x))$. First, if we want to usefully write $m(x) = f(g(x))$, then
$$f(u) = \qquad\qquad\qquad \text{and} \qquad g(x) =$$
so (using which rule to find $g'(x)$?)
$$f'(u) = \qquad\qquad\qquad \text{and} \qquad g'(x) =$$
Now, if $m(x) = \cos(x\sin(x))$ then $m'(x) =$

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