WORKSHEET: CHAIN RULE

Names and student IDs: ____

Chain rule: If g is differentiable at x and f is differentiable at $g(x)$, and if

$$h(x) = f(g(x))$$

for all x (in a suitable open interval), then

$$h'(x) = f'(g(x)) \cdot g'(x).$$

Example for the chain rule: If $h(x) = \sin(x^3)$ then write $h(x) = f(g(x))$ with

$$f(u) = \sin(u) \qquad \text{and} \qquad g(x) = x^3.$$

Thus

$$h'(x) = f'(g(x)) \cdot g'(x) = \sin'(x^3) \cdot \frac{d}{dx}\big(x^3\big) = \cos(x^3) \cdot 3x^2 = 3x^2 \cos(x^3).$$

The last step is conventional. Caution: "$\sin'(x^3)$" means you take the derivative of the sine function and evaluate it at x^3. It does *not* mean ~~$\frac{d}{dx}\big(\sin(x^3)\big)$~~. The expression $\frac{d}{dx}\big(\sin(x^3)\big)$ means $h'(x)$, the derivative with respect to x of the function whose value at x is $\sin(x^3)$.

Now differentiate the following functions, identifying the appropriate choices of f and g in the formula above. (If we don't get to these in class, do them at home.)

Let $w(x) = \cos(x^4)$. First, if we want to usefully write $w(x) = f(g(x))$, then

$f(u) =$ and $g(x) =$

Now, if $w(x) = \cos(x^4)$ then $w'(x) =$

Let $p(x) = (x^{11} + 3x + 1)^{109}$. First, if we want to usefully write $p(x) = f(g(x))$, then

$f(u) =$ and $g(x) =$

Now, $\frac{d}{dx}\big((x^{11} + 3x + 1)^{109}\big) =$

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