WORKSHEET SOLUTIONS: PRODUCT AND QUOTIENT RULES

Names and student IDs: Solutions [$\pi\pi\pi$-$\pi\pi$-$\pi\pi\pi\pi$]

Recall the product rule: if f and g are differentiable for all x in a suitable open interval, then

$$\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x).$$

Also recall that, using **radians**,

$$\sin'(x) = \cos(x) \qquad \text{and} \qquad \cos'(x) = -\sin(x)$$

for all real x.

Use the product rule to differentiate the following functions.

1. $q(x) = 3x^7\cos(x)$.

Solution. Write $q(x) = f(x)g(x)$ with

$$f(x) = 3x^7 \qquad \text{and} \qquad g(x) = \cos(x).$$

Thus:

$$\begin{aligned} q'(x) = f'(x)g(x) + f(x)g'(x) &= \frac{d}{dx}\big(3x^7\big)\cos(x) + 3x^7\cos'(x) \\ &= 21x^6\cos(x) + 3x^7(-\sin(x)) = 21x^6\cos(x) - 3x^7\sin(x). \end{aligned}$$

□

2. $P(t) = (27t^{10} + 3t - 8)(11t^6 - 101t^4)$. (Don't multiply anything out.)

Solution.

$$\begin{aligned} P'(t) &= \frac{d}{dt}\big(27t^{10} + 3t - 8\big)(11t^6 - 101t^4) + (27t^{10} + 3t - 8)\frac{d}{dt}\big(11t^6 - 101t^4\big) \\ &= (270t^9 + 3)(11t^6 - 101t^4) + (27t^{10} + 3t + 8)(66t^5 - 404t^3\big). \end{aligned}$$

□

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