WORKSHEET: DERIVATIVES FROM THE DEFINITION

Names and student IDs: _____

1. Find $\lim_{x\to 3} \dfrac{x^2 - x - 6}{2x^2 + x + 7}$.

2. Find $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x^2 + x - 12}$.

Recall: if f is a function defined on an open interval containing a, then the derivative of f at a is $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$, **if this limit exists**. (An alternate formulation is: the derivative of f at a is $f'(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$, **if this limit exists**.)

3. Let $k(x) = x^2$ for all real numbers x. Find $k'(3)$ directly from the definition. (You should get $k'(3) = 6$.)

4. Let k be as above. Let a be an arbitrary real number. Find $k'(a)$ directly from the definition. (You should get $k'(a) = 2a$.)

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5. If $k(x) = x^2$ for all real numbers x, then for all real numbers x, we have $k'(x) =$ ____.

Do the remaining problems at home after class if we run out of time.

6. Expand the expression $(x + h)^3$.

7. Let $l(x) = x^3$ for all real numbers x. Let a be an arbitrary real number. Find $l'(x)$ directly from the definition. You should get $l'(x) = 3x^2$.

8. For a nonnegative integer n, let $P_n(x)$ be the function $P_n(x) = x^n$ for all real numbers x. (Take $P_0(0) = 1$.)

You have seen $P_0'(x)$, $P_1'(x)$, $P_2'(x)$, and $P_3'(x)$. What do you think $P_4'(x)$ is? What do you think $P_n'(x)$ is?