

WORKSHEET: THE FALLING CALCULUS BOOK

Names and student IDs: _____

The gravitational constant at the surface of the planet Yuggxth is exactly 2 m/sec^2 . It has no atmosphere, so falling objects encounter no friction. A calculus book is dropped down a vertical hole at time $t = 0 \text{ sec}$, so its height at time $t \text{ sec}$ is $p(t) = -t^2 \text{ m}$ above the surface. (Nobody on Yuggxth knows any calculus.) Note: $-t^2$, because the calculus book is falling *down*.

We want to know the velocity of the calculus book at time $t = 2 \text{ sec}$. (This is analogous to the reading of the speedometer in a car.)

As a first approximation, let's find the average velocity over the time interval $[2, 4]$ (in seconds). It is

$$\frac{p(4) - p(2)}{4 - 2} = \frac{-4^2 - (-2^2)}{4 - 2} = \frac{-12}{2} = -6 \text{ m/sec}.$$

The answer is negative because the calculus book is moving down and we measured height in meters *above* the surface. Thus, the calculus book is rising at -6 m/sec , or falling at 6 m/sec . Keep the signs right!

1. Use the same method to find the average velocity over the time interval $[2, 3]$ (in seconds).

2. Use the same method to find the average velocity over the time interval $[2, 2.1]$ (in seconds). (Feel free to use a calculator on this one.)

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3. Use the same method to find the average velocity over the time interval $[2, 2.01]$ (in seconds). (Feel free to use a calculator on this one.)

4. Use the same method to find the average velocity over the time interval $[1, 2]$ (in seconds).

5. Use the same method to find the average velocity over the time interval $[1.9, 2]$ (in seconds). (Feel free to use a calculator on this one.)

6. What goes wrong if you try to find the velocity at $t = 2$ sec by taking both times to be $t = 2$ sec?

7. Nevertheless, what do you think the velocity at $t = 2$ sec actually is?