Math 251   Midterm 2   NAME:_____

F 23 May 2025.   Student ID:_____

<u>GENERAL INSTRUCTIONS</u>

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

2. The exam pages are **two sided**.

3. Closed book, except for a 3×5 file card, written on both sides.

4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

5. The point values are as indicated in each problem; total 100 points.

6. Write all answers on the test paper. Use the back of the last page page for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)

7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.

8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. (Cross out any work you do not want to have considered.) Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.

9. Be sure you say what you mean. Credit will be based on what you say, not what you mean.

10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.

11. Final answers must always be simplified unless otherwise specified.

12. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

13. Time: 50 minutes.

1	2	3	4	5	6	TOTAL	EC
1	12	20	27	20	20	100	

1. (1 point.) True or false: Related rates problems were invented by the devil.

2. (12 points) Find the exact value of the limit $\lim_{x \to 4^+} \dfrac{x - 8}{x - 4}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

3. (20 points) If $\sin(y) = (4x - y)^3 - \ln(x)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

4. (27 points) A box shaped storage shed will have a square base, square roof, and rectangular sides, and total volume 18 cubic yards. The floor will be just bare ground, and one of the sides will be open. What height minimizes the total area of the roof and the other three sides?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

5. (20 points.) A magical square field has a side length which varies with time. On 29 February 2025, its sides were 20 meters long, and were increasing at the rate of 3 meters per week. At this time, was its area increasing or decreasing? How fast? (Be sure to include the correct units.)

6. (20 points) Let $c(x) = x^3 - 3x^2 - 24x + 5$. Identify the open intervals on which c is increasing, those on which c is decreasing, and all critical points, local minimums, and local maximums.

Extra credit. (15 extra credit points; grading will be harsher than on related problems on the main exam. Do not attempt this problem until you have done and checked your answer to all the ordinary problems on this exam. It will only be counted if you get a grade of B or better on the main part of this exam.)

A four dimensional box has a cubical base and no top. Its four dimensional volume is supposed to be $8\,\mathrm{ft}^4$. What dimensions minimize the three dimensional volume of material needed to make its base and sides?

Hint: A box in four dimensional space has 8 three dimensional "faces", each of which has the shape of a three dimensional box.

(Use the back of the page if needed.)