SOLUTIONS TO THE SAMPLE MIDTERM 2, MATH 251 (PHILLIPS), SPRING 2025

Contents

1. Midterm 2 Information

At least 80% of the points on the real exam will be modifications of problems from Midterms 1 and 2 from the last time I taught the course, midterms and quizzes so far in this course (including Midterms 0), the problems below, homework problems (including written homework and WeBWorK), and problems from the sample and real Midterms 0. Note, though, that the exact form of the functions to be differentiated and of the limits to be computed could vary substantially, and the methods required to do them might occur in different combinations. Word problems could have rather different descriptions, but similar methods will be used.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation for limits etc. in the book, in handouts, in files posted on the course website, and on the blackboard; *use it*. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Here is the instruction sheet for Midterm 2. There is a slight update.

(1) DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
(2) The exam pages are **two sided**.
(3) Closed book, except for a 3×5 file card, written on both sides.
(4) The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
(5) The point values are as indicated in each problem; total 100 points.
(6) Write all answers on the test paper. Use the bottom of page 5 for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
(7) Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.

(8) Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Cross out any work you do not want considered. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
(9) Be sure you say what you mean. Credit will be based on what you say, not what you mean.
(10) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
(11) Final answers must always be simplified unless otherwise specified.
(12) Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
(13) Time: 50 minutes.

2. Sample Midterm 2

1. (25 points.) A cylindrical water tank, with a circular base, is to have a volume of 40π cubic feet. The material for the side costs \$10 per square foot, the material for the bottom costs \$30 per square foot, and the material for the top costs \$20 per square foot. What is the minimum possible cost of the materials for such a tank?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

Solution: Note: There is no picture in this file. A picture may be provided separately.

We want to minimize the cost of the materials for the bin; call it C.

Let h be the height of the bin, and let r be the radius of its base, both measured in feet. The area of the bottom is πr^2 (in square feet), so it costs $30\pi r^2$ (dollars). The area of the top is also πr^2, so it costs $20\pi r^2$. The area of the side is the circumference times the height, or $2\pi r h$. So it costs $10 \cdot 2\pi r h$. Thus

$$C = 30\pi r^2 + 20\pi r^2 + 10 \cdot 2\pi r h = 50\pi r^2 + 20\pi r h. = 10\big(5\pi r^2 + 2\pi r h\big).$$

There are too many variables here, and we can eliminate one of them using the constraint on the volume V. The volume is the height times the area of the base, that is, $V = \pi r^2 h$, and we are told it is supposed to be $V = 40\pi$. So $\pi r^2 h = 40\pi$, and $r^2 h = 40$.

It is easier to solve for h than for r, so we do that, giving $h = 40r^{-2}$. Substituting in the formula for C, and writing the result as a function for the purposes of differentiation, we get

$$C(r) = 10\big(5\pi r^2 + 2\pi r \cdot 40r^{-2}\big) = 10\left(5\pi r^2 + \frac{80\pi}{r}\right) = 10\pi\left(5r^2 + \frac{80}{r}\right).$$

Note that r must be positive ($r > 0$). Also, we must have $h > 0$, but, since $h = 40r^{-2}$, this does not tell us anything new. So we are to minimize $C(r) = 10\pi\left(5\pi r^2 + \frac{80\pi}{r}\right)$ subject to the condition $r > 0$.

We start by searching for critical numbers. We differentiate: $C'(r) = 10\pi\big(10r - 80r^{-2}\big)$. We equate this to zero:

$$\begin{aligned} 10\pi\big(10r - 80r^{-2}\big) &= 0 \\ 10r - 80r^{-2} &= 0 \\ r - \frac{8}{r^2} &= 0 \end{aligned}$$

$$r^3 - 8 = 0$$
$$r = 2.$$

So $r = 2$ is the only critical number.

Is it a maximum or minimum? The easiest test to use here is the second derivative test. One easily checks that $C''(r) = 10\pi\big(10 + 160r^{-3}\big)$, which is positive for all r in the interval $(0, \infty)$. Therefore C is concave up everywhere on the interval $(0, \infty)$, and any critical number must be an absolute minimum.

The problem asked for the minimum cost. This is now $C(2) = 10\pi\left(5 \cdot 2^2 + \frac{80}{2}\right) = 600\pi$ dollars. (Don't forget the units!)

For those who tried it instead of the second derivative test as done above, here is how the first derivative test works. We factor the derivative as follows:

$$C'(r) = 10\pi\big(10r - 80r^{-2}\big) = 100\pi r^{-2}(r^3 - 8).$$

For $r > 0$, all factors except the last are positive. So we need only consider the sign of $r^3 - 8$. For $0 < r < 2$, we have $r^3 - 8 < 0$, so $C'(r) < 0$. Therefore C is decreasing on the whole interval $(0, 2)$. For $r > 2$, we have $r^3 - 8 > 0$, so $C'(r) > 0$. Therefore C is increasing on the whole interval $(2, \infty)$. Clearly, then, there is an absolute minimum at $r = 2$.

Here is a different approach, perhaps simpler, to the first derivative test. We know that $C'(r)$ is continuous on the whole interval $(0, \infty)$. It is zero only at $r = 2$. Therefore, by the Intermediate Value Theorem, the derivative must have the same sign throughout the interval $(0, 2)$, and we can find that sign by computing, say, $C'(1) = -700\pi < 0$. The derivative must also have the same sign throughout the interval $(2, \infty)$, and we get it by computing, say, $C'(4) = 100\pi \cdot 4^{-2}(4^3 - 8) = \frac{5600}{16}\pi > 0$. As above, then, $C(r)$ is decreasing on the whole interval $(0, 2)$ and increasing on the whole interval $(2, \infty)$, so has an absolute minimum at $r = 2$.

Finally, here is the test using limits at the ends of the domain. We compare $C(2) = 600\pi$ (computed above),

$$\lim_{r \to 0} C(r) = \lim_{r \to 0} 10\pi\left(5r^2 + \frac{80}{r}\right) = \infty,$$

and

$$\lim_{r \to \infty} C(r) = \lim_{r \to \infty} 10\pi\left(5r^2 + \frac{80}{r}\right) = \infty.$$

Obviously 600π is the smallest of these.

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2. (20 points) A certain section of the San Andreas Fault runs straight north-south. On 29 February 1996, the west side was moving north (relative to the east side) at 3 cm/year (0.03 meters/year). At the same time, the town of Hicksville was 1 km (1000 meters) west of the fault, and the town of Quoggin was 2 km (2000 meters) east of the fault and 4 km (4000 meters) farther north than Hicksville. Were these two towns getting closer together or farther apart at this time? At what rate?

Solution: Note: There is no picture in this file. A picture may be provided separately.

The route from Hicksville to Quoggin at the present time, as described in the problem, is a zigzag line, first going east 1000 meters to the fault, then north 4000 meters along the fault, then east 2000 meters from the fault to Quoggin. Both the east-west distances are constant, but the north-south distance along the fault is actually changing. So let's call it $y(t)$, and say that $t = 0$ is the present time. Thus $y(0) = 4000$ (measuring all distances in meters). A more appropriate description of the route from Hicksville to Quoggin, valid at an arbitrary time, is therefore a zigzag line which first goes east 1000 meters to the fault, then north $y(t)$ meters along the fault, then east 2000 meters from the fault to Quoggin.

To find the distance, we are better off considering the route which starts at Hicksville, goes 3000 meters east, and then goes $y(t)$ meters north to Quoggin. These are two sides of a right triangle, so the distance $l(t)$ in meters from Hicksville to Quoggin at time t is

$$l(t) = \sqrt{3000^2 + y(t)^2} = \left(3000^2 + y(t)^2\right)^{1/2}.$$

We want to find $l'(0)$. (Note: it is less direct, but easier, to differentiate the equation $l(t)^2 = 3000^2 + y(t)^2$. See below.)

Differentiating, we get

$$l'(t) = \tfrac{1}{2}\left(3000^2 + y(t)^2\right)^{-1/2} \cdot 2y(t)y'(t) = y(t)y'(t)\left(3000^2 + y(t)^2\right)^{-1/2}.$$

(Don't forget to use the chain rule!) Put $t = 0$ and substitute values. (This can only be done *after* differentiating!) We know $y(0) = 4000$. We need $y'(0)$, which we can see from the statement is -0.03. (It is negative because $y(t)$ is decreasing.) So

$$l'(0) = y(0)y'(0)\left(3000^2 + y(0)^2\right)^{-1/2} = 4000 \cdot (-0.03) \cdot \tfrac{1}{5000} = -0.024.$$

Therefore Hicksville and Quoggin are getting closer together at 0.024 meters per year, or 2.4 cm/year. (The units are necessary!)

Some people in Quoggin consider this to be bad news. (So do some people in Hicksville.)

Here are descriptions of some alternatives. First, you could differentiate the equation $l(t)^2 = 3000^2 + y(t)^2$, getting

$$2l(t)l'(t) = 2y(t)y'(t),$$

so that

$$l'(t) = \frac{y(t)y'(t)}{l(t)}.$$

Now put $t = 0$, and substitute $y(0) = 4000$, $y'(0) = -0.03$, and $l(0) = 5000$. (You still need to calculate $l(0)$ from the Pythagorean Theorem.)

You could also do everything in physicists' notation. I will only show the first version. The equation for l is

$$l = \sqrt{3000^2 + y^2} = \left(3000^2 + y^2\right)^{1/2}.$$

Differentiating (using the chain rule, because everything is a function of t!), we get

$$\frac{dl}{dt} = \tfrac{1}{2}\left(3000^2 + y^2\right)^{-1/2} \cdot 2y\frac{dy}{dt} = y\frac{dy}{dt}\left(3000^2 + y^2\right)^{-1/2}.$$

Substituting values (implicitly putting $t = 0$, and using $l = 5000$ at $t = 0$, as above):

$$\frac{dl}{dt} = y\frac{dy}{dt}\left(3000^2 + y^2\right)^{-1/2} = 4000 \cdot (-0.03) \cdot \tfrac{1}{5000} = -0.024,$$

as before.

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3. (15 points) If $2x + \arcsin(y) = (3x + y)^5 + \cos(11)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

Solution: Differentiate both sides with respect to x, using the chain rule on both sides:

$$2 + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 5(3x+y)^4 \frac{d}{dx}(3x+y) = 5(3x+y)^4\left(3 + \frac{dy}{dx}\right).$$

(The derivative of $\cos(11)$ is immediately seen to be zero because $\cos(11)$ is a constant.) Now solve for $\frac{dy}{dx}$:

$$2 + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 15(3x+y)^4 + 5(3x+y)^4\frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} - 5(3x+y)^4\frac{dy}{dx} = 15(3x+y)^4 - 2$$

$$\left[\frac{1}{\sqrt{1-y^2}} - 5(3x+y)^4\right]\frac{dy}{dx} = 15(3x+y)^4 - 2$$

$$\frac{dy}{dx} = \frac{15(3x+y)^4 - 2}{\frac{1}{\sqrt{1-y^2}} - 5(3x+y)^4}.$$

This fraction can't be simplified.

For those who prefer the other notation, here it is written with y as an explicit function $y(x)$ of x. Start with

$$2x + \arcsin(y(x)) = [3x + y(x)]^5 + \cos(11).$$

Then differentiate with respect to x, just as before:

$$2 + \frac{y'(x)}{\sqrt{1-y^2}} = 5[3x+y(x)]^4\frac{d}{dx}(3x+y(x)) = 5[3x+y(x)]^4(3+y'(x)).$$

Now solve for $y'(x)$:

$$2 + \frac{y'(x)}{\sqrt{1-y^2}} = 15[3x+y(x)]^4 + 5[3x+y(x)]^4 y'(x)$$

$$\left[\frac{1}{\sqrt{1-y^2}}\right]y'(x) - 5[3x+y(x)]^4 y'(x) = 15[3x+y(x)]^4 - 2$$

$$\left[\frac{1}{\sqrt{1-y^2}} - 5[3x+y(x)]^4\right]y'(x) = 15[3x+y(x)]^4 - 2$$

$$y'(x) = \frac{15[3x+y(x)]^4 - 2}{\frac{1}{\sqrt{1-y^2}} - 5[3x+y(x)]^4}.$$

As before, this fraction can't be simplified.

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4. (10 points) Differentiate the function $y = \frac{x}{\arctan(kx)} + \frac{\sqrt{\pi}}{2}$, where k is a constant.

Solution: Use the quotient rule. For $\dfrac{d}{dx}(\arctan(kx))$, it is necessary to use the chain rule, noting that, since k is a constant, we have $\dfrac{d}{dx}(kx) = k$. Thus

$$\begin{aligned}
\frac{dy}{dx} &= \frac{\frac{d}{dx}(x) \cdot \arctan(kx) - x\frac{d}{dx}(\arctan(kx))}{[\arctan(kx)]^2} \\
&= \frac{\arctan(kx) - x \cdot \frac{1}{1+(kx)^2}\frac{d}{dx}(kx)}{[\arctan(kx)]^2} \\
&= \frac{\arctan(kx) - \frac{kx}{1+(kx)^2}}{[\arctan(kx)]^2}.
\end{aligned}$$

The derivative of $\dfrac{\sqrt{\pi}}{2}$ is zero because $\dfrac{\sqrt{\pi}}{2}$ is a constant. (Don't waste time using the quotient rule to differentiate $\dfrac{\sqrt{\pi}}{2}$!)

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5. (10 points) Let $f(x)$ be the function given by $f(x) = (x^3 + 3x^2 + 3x + 5)e^{-x}$. Its derivative is given by $f'(x) = -(x+2)(x-1)^2e^{-x}$. (You **need not** check this.) Find the critical numbers of $f(x)$, and for each one determine if it is a local minimum, local maximum, or neither.

Solution: Since e^{-x} is never zero, we have $f'(x) = 0$ exactly when $x = -2$ or $x = 1$. The critical numbers are thus -2 and 1.

To test whether these are local minimums or local maximums, we determine the intervals on which f is increasing and the intervals on which f is decreasing.

For $x < -2$,

$$x + 2 < 0, \qquad (x-1)^2 > 0, \qquad \text{and} \qquad e^{-x} > 0.$$

Therefore

$$f'(x) = -(x+2)(x-1)^2e^{-x} > 0,$$

and it follows that f is increasing on the interval $(-\infty, -2)$.

When $-2 < x < 1$,

$$x + 2 > 0, \qquad (x-1)^2 > 0, \qquad \text{and} \qquad e^{-x} > 0.$$

Therefore

$$f'(x) = -(x+2)(x-1)^2e^{-x} < 0,$$

and it follows that f is decreasing on the interval $(-2, 1)$.

For $x > 1$,

$$x + 2 > 0, \qquad (x-1)^2 > 0, \qquad \text{and} \qquad e^{-x} > 0.$$

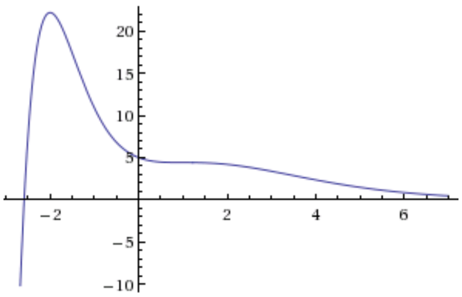
Therefore

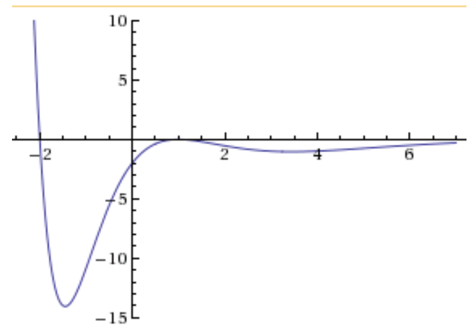
$$f'(x) = -(x+2)(x-1)^2e^{-x} < 0,$$

and it follows that f is decreasing on the interval $(1, \infty)$.

Since f is increasing on $(-\infty, -2)$ and decreasing on $(-2, 1)$, it follows that f has a local maximum at -2. Since f is decreasing on $(-2, 1)$ and also decreasing on $(1, \infty)$, it follows that f has neither a local minimum nor a local maximum at 1.

For reference, here are graphs of f (below left) and f' (below right). (These are not required as part of the solution.)





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6. (10 points) Find the exact value of the limit $\lim_{x \to 3^-} \frac{x^2 - 4}{x - 3}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution: At 3, the denominator is zero but the numerator is not. Moreover, both the denominator and the numerator are continuous at 3. Therefore the function $f(x) = \frac{x^2-4}{x-3}$ has a vertical asymptote at $x = 3$.

For $x < 3$ but very close to 3, the denominator is negative and very close to zero. The numerator is very close to $3^2 - 4 = 5$, so is positive and not close to zero. Therefore the quotient is negative and very far from zero. Thus

$$\lim_{x \to 3^-} \frac{x^2 - 4}{x - 3} = -\infty.$$

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7. (10 points) Find the exact value of the limit $\lim_{x \to -\infty} \left(2 + \frac{1}{x} + 5x^2 \right)$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution:

We have (showing more steps than necessary)

$$\lim_{x \to -\infty} 2 = 2 \qquad \text{and} \qquad \lim_{x \to -\infty} \frac{1}{x} = 0.$$

Also, $\lim_{x \to -\infty} x^2 = \infty$ (note: not ~~$-\infty$~~), so $\lim_{x \to -\infty} 5x^2 = \infty$. Since the limits of the other two summands are finite, it follows that

$$\lim_{x \to -\infty} \left(2 + \frac{1}{x} + 5x^2 \right) = \infty.$$

Since ∞ and $-\infty$ are not numbers, we can't do arithmetic on them. (In this case, the correct answer is as if you could.) Therefore, any solution containing

~~∞^2~~, ~~$(-\infty)^2$~~, or ~~$2 + \infty$~~

uses incorrect notation, and will therefore lose points.

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3. Extra Sample Problems for Midterm 2

The limit problems are here to illustrate some of the different possible outcomes for such problems.

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8. (8 points) A function g defined and differentiable on $(-\infty, \infty)$, satisfies $g'(-4) = g'(3) = 0$, $g'(x) > 0$ on $(-\infty, -4)$, $g'(x) < 0$ on $(-4, 3)$, and $g'(x) > 0$ on $(3, \infty)$. Identify the open intervals on which g is increasing, those on which g is decreasing, and all critical points, local minimums, and local maximums.

Solution: The problem tells us directly that the critical points are -4 and 3. The information on the sign of $g'(x)$ tells you that g is increasing on $(-\infty, \infty)$ and on $(3, \infty)$, and decreasing on $(-4, 3)$. Therefore g has a local maximum at -4 and a local minimum at 3.

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9. (10 points) Find the exact value of the limit $\lim_{x\to\infty} \dfrac{\arctan(x)}{1 + x^{-1}}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution: Since $\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$ and $\lim_{x\to\infty}(1 + x^{-1}) = 1 \neq 0$, we have

$$\lim_{x\to\infty} \frac{\arctan(x)}{1 + x} = \frac{\lim_{x\to\infty} \arctan(x)}{\lim_{x\to\infty}(1 + x^{-1})} = \frac{\pi/2}{1} = \frac{\pi}{2}.$$

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10. (10 points) Find the exact value of the limit $\lim_{x\to 4} \dfrac{x^2 - 4}{x - 4}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution: At 4, the denominator is zero but the numerator is not. Moreover, both the denominator and the numerator are continuous at 4. Therefore the function $f(x) = \frac{x^2-4}{x-4}$ has a vertical asymptote at $x = 4$.

For $x < 4$ but very close to 4, the denominator is negative and very close to zero. The numerator is very close to $4^2 - 4 = 12$, so is positive and not close to zero. Therefore the quotient is negative and very far from zero. Thus

$$\lim_{x\to 4^-} \frac{x^2 - 4}{x - 4} = -\infty.$$

Similarly,

$$\lim_{x\to 4^+} \frac{x^2 - 4}{x - 4} = \infty.$$

Therefore $\lim_{x\to 4} \dfrac{x^2 - 4}{x - 4}$ does not exist, and is also not ∞ or $-\infty$.

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11. (10 points) Find the exact value of the limit $\lim_{x\to 2} \dfrac{x^2 - 4}{x^2 + x - 6}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution: This limit has the indeterminate form "$\frac{0}{0}$", so work is needed. We factor the denominator, and then cancel common factors in the fraction:

$$\lim_{x\to 2} \frac{x^2-4}{x^2+x-6} = \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x+3)} = \lim_{x\to 2} \frac{x+2}{x+3} = \frac{2+2}{2+3} = \frac{4}{5}.$$

Any solution containing

$$\cancel{=\frac{0}{0}}, \qquad \cancel{\lim_{x\to 2} =}, \qquad \text{or} \qquad \cancel{\lim_{x\to 2} \frac{4}{5}}$$

uses incorrect notation, and will therefore lose points.

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12. (10 points) Find the exact value of the limit $\lim_{x\to\infty} \left(3 + \frac{7}{x^3}\right)$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

Solution: We have (showing more steps than necessary)

$$\lim_{x\to\infty} \left(3 + \frac{7}{x^3}\right) = \lim_{x\to\infty} 3 + 7 \lim_{x\to\infty} \frac{1}{x^3} = 3 + 7\left(\lim_{x\to\infty} \frac{1}{x}\right)^3 = 3 + 7\cdot 0^3 = 3.$$

Any solution containing

$$\cancel{=\frac{7}{\infty}} \qquad \text{or} \qquad \cancel{\infty^3}$$

uses incorrect notation, and will therefore lose points.

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13. (10 points) Let $h(x) = \dfrac{\ln\left(7x + \sqrt{2\pi}\right)}{2x} + \dfrac{1}{5}$. Find $h'(x)$.

Solution: The second term is a constant, so its derivative is zero. We use the quotient rule on the first term, using the chain rule its numerator. This gives

$$h'(x) = \frac{\left(\frac{1}{7x+\sqrt{2\pi}}\right)(7)(2x) - \ln\left(7x+\sqrt{2\pi}\right)\cdot 2}{(2x)^2} = \frac{2\left(\frac{7x}{7x+\sqrt{2\pi}}\right) - 2\ln\left(7x+\sqrt{2\pi}\right)}{4x^2}$$
$$= \frac{\frac{7x}{7x+\sqrt{2\pi}} - \ln\left(7x+\sqrt{2\pi}\right)}{2x^2}.$$

There is no sensible further simplification, but the simplification that has been done (cancelling the factors of 2) is required.

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14. (10 points) Find $g'(2)$, where $g = f^{-1}$ is the inverse of the function

$$f(x) = \sqrt[3]{x^7 + 3x + 4}.$$

(Hint: $f(1) = 2$.)

Solution: We have

$$g'(x) = (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

(You can get this from the chain rule by differentiating the equation $f(f^{-1}(x)) = x$.)

Now put $x = 2$ and note that $f^{-1}(2) = 1$. So

$$g'(2) = (f^{-1})'(2) = \frac{1}{f'(1)}.$$

So we need to calculate $f'(x)$. We have

$$f(x) = \left(x^7 + 3x + 4\right)^{1/3},$$

so

$$f'(x) = \tfrac{1}{3}\left(x^7 + 3x + 4\right)^{-2/3} \cdot (7x^6 + 3),$$

and

$$f'(1) = \tfrac{1}{3}\left(1^7 + 3 \cdot 1 + 4\right)^{-2/3} \cdot (7 \cdot 1^6 + 3) = \tfrac{1}{3} \cdot 8^{-2/3} \cdot 10 = \tfrac{1}{3} \cdot \tfrac{1}{4} \cdot 10 = \tfrac{5}{6}.$$

Therefore $g'(2) = \frac{6}{5}$.
