SAMPLE MIDTERM 2, MATH 251 (PHILLIPS), SPRING 2025

Contents

1. Midterm 2 Information

At least 80% of the points on the real exam will be modifications of problems from Midterms 1 and 2 from the last time I taught the course, midterms and quizzes so far in this course (including Midterms 0), the problems below, homework problems (including written homework and WeBWorK), and problems from the sample and real Midterms 0. Note, though, that the exact form of the functions to be differentiated and of the limits to be computed could vary substantially, and the methods required to do them might occur in different combinations. Word problems could have rather different descriptions, but similar methods will be used.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation for limits etc. in the book, in handouts, in files posted on the course website, and on the blackboard; *use it*. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Here is the instruction sheet for Midterm 2. There is a slight update.

(1) DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
(2) The exam pages are **two sided**.
(3) Closed book, except for a 3×5 file card, written on both sides.
(4) The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
(5) The point values are as indicated in each problem; total 100 points.
(6) Write all answers on the test paper. Use the bottom of page 5 for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
(7) Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.

(8) Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. Cross out any work you do not want considered. Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.
(9) Be sure you say what you mean. Credit will be based on what you say, not what you mean.
(10) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
(11) Final answers must always be simplified unless otherwise specified.
(12) Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
(13) Time: 50 minutes.

2. Sample Midterm 2

1. (25 points.) A cylindrical water tank, with a circular base, is to have a volume of 40π cubic feet. The material for the side costs \$10 per square foot, the material for the bottom costs \$30 per square foot, and the material for the top costs \$20 per square foot. What is the minimum possible cost of the materials for such a tank?

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

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2. (20 points) A certain section of the San Andreas Fault runs straight north-south. On 29 February 1996, the west side was moving north (relative to the east side) at 3 cm/year (0.03 meters/year). At the same time, the town of Hicksville was 1 km (1000 meters) west of the fault, and the town of Quoggin was 2 km (2000 meters) east of the fault and 4 km (4000 meters) farther north than Hicksville. Were these two towns getting closer together or farther apart at this time? At what rate?

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3. (15 points) If $2x + \arcsin(y) = (3x + y)^5 + \cos(11)$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

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4. (10 points) Differentiate the function $y = \dfrac{x}{\arctan(kx)} + \dfrac{\sqrt{\pi}}{2}$, where k is a constant.

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5. (10 points) Let $f(x)$ be the function given by $f(x) = (x^3 + 3x^2 + 3x + 5)e^{-x}$. Its derivative is given by $f'(x) = -(x+2)(x-1)^2e^{-x}$. (You **need not** check this.) Find the critical numbers of $f(x)$, and for each one determine if it is a local minimum, local maximum, or neither.

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6. (10 points) Find the exact value of the limit $\lim\limits_{x \to 3^-} \dfrac{x^2 - 4}{x - 3}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

7. (10 points) Find the exact value of the limit $\lim_{x\to-\infty}\left(2+\frac{1}{x}+5x^2\right)$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

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3. Extra Sample Problems for Midterm 2

The limit problems are here to illustrate some of the different possible outcomes for such problems.

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8. (8 points) A function g defined and differentiable on $(-\infty,\infty)$, satisfies $g'(-4)=g'(3)=0$, $g'(x)>0$ on $(-\infty,-4)$, $g'(x)<0$ on $(-4,3)$, and $g'(x)>0$ on $(3,\infty)$. Identify the open intervals on which g is increasing, those on which g is decreasing, and all critical points, local minimums, and local maximums.

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9. (10 points) Find the exact value of the limit $\lim_{x\to\infty}\frac{\arctan(x)}{1+x^{-1}}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

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10. (10 points) Find the exact value of the limit $\lim_{x\to4}\frac{x^2-4}{x-4}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

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11. (10 points) Find the exact value of the limit $\lim_{x\to2}\frac{x^2-4}{x^2+x-6}$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

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12. (10 points) Find the exact value of the limit $\lim_{x\to\infty}\left(3+\frac{7}{x^3}\right)$ (possibly ∞ or $-\infty$), or explain why it does not exist. Give reasons.

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13. (10 points) Let $h(x)=\dfrac{\ln\left(7x+\sqrt{2\pi}\right)}{2x}+\dfrac{1}{5}$. Find $h'(x)$.

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14. (10 points) Find $g'(2)$, where $g=f^{-1}$ is the inverse of the function

$$f(x)=\sqrt[3]{x^7+3x+4}.$$

(Hint: $f(1)=2$.)
