Math 251   Midterm 1   NAME:_____

F 25 April 2025.   Student ID:_____

<u>GENERAL INSTRUCTIONS</u>

1. DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

2. The exam pages are **two sided**.

3. Closed book, except for a 3×5 file card, written on both sides.

4. The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.

5. The point values are as indicated in each problem; total 100 points.

6. Write all answers on the test paper. Use the back of the last page for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)

7. Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting "=", limit symbols, etc. in all places where they belong, and not in any places where they don't belong, as well as using all required parentheses. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.

8. Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. (Cross out any work you do not want to have considered.) Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.

9. Say what you mean. Credit will be based on what you say, not what you mean.

10. When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.

11. Final answers must always be simplified unless otherwise specified.

12. Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).

13. Time: 50 minutes. (unless extended, with class agreement, by an early start).

1	2	3	4	5	6	7	8	9	10	TOTAL	EC
1	6	11	11	11	7	16	15	10	12	100	

1. (1 point) Are you awake?

2. (6 points) State carefully the definition of the derivative of a function.

3. (a) (10 points) If $f(x) = x^2 + 3$, compute the derivative $f'(4)$ *directly from the definition of the derivative* (which you are supposed to have given above). (No credit will be given for just using the differentiation rules, but see Part (b.)

   (b) (1 point) Use the differentiation rules we have learned to check your answer to part (a).

4. (11 points) Differentiate the function $q(x) = (2x^4 - 11x + 8)\sin(x) + \dfrac{1}{15}$. (You need not do this directly from the definition.)

5. (11 points) Differentiate the function $h(x) = \cos(4x^3 + 8x)$. (You need not do this directly from the definition.)

6. (7 points) Find the exact value of the limit $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{2x^2 + x + 7}$, or explain why this limit does not exist.

7. (16 points.) A farmer wants to build a rectangular fenced enclosure. Because of bizarre local laws, the east fence will cost 7 florins per meter, the west fence will cost 3 florins per meter, the north fence will cost 4 florins per meter, and the south fence will cost 2 florins per meter. The farmer has 6000 florins available to build the enclosure. The farmer wants to find the lengths of the south and west fences of the enclosure with the largest area that can be built.

Set up, but **do not attempt to solve**, the appropriate maximization or minimization problem. That is, give a function $f(x)$, for x a suitable quantity related to the problem (say what x actually is!), give a suitable domain, and say whether you want to maximize or minimize f on this domain. Provide justification for all steps (possibly including a picture).

8. (15 points) Use the methods of calculus to find the exact values of x at which the function $k(x) = x^3 - 6x^2 - 15x$ takes its absolute minimum and maximum on the interval $[-2, 2]$.

(No credit will be given for correct guesses without supporting work that is valid for general functions of the sort considered in this course.)

9. (10 points.) This problem is about using correct notation. Accordingly, almost all the credit is for correctness of notation.
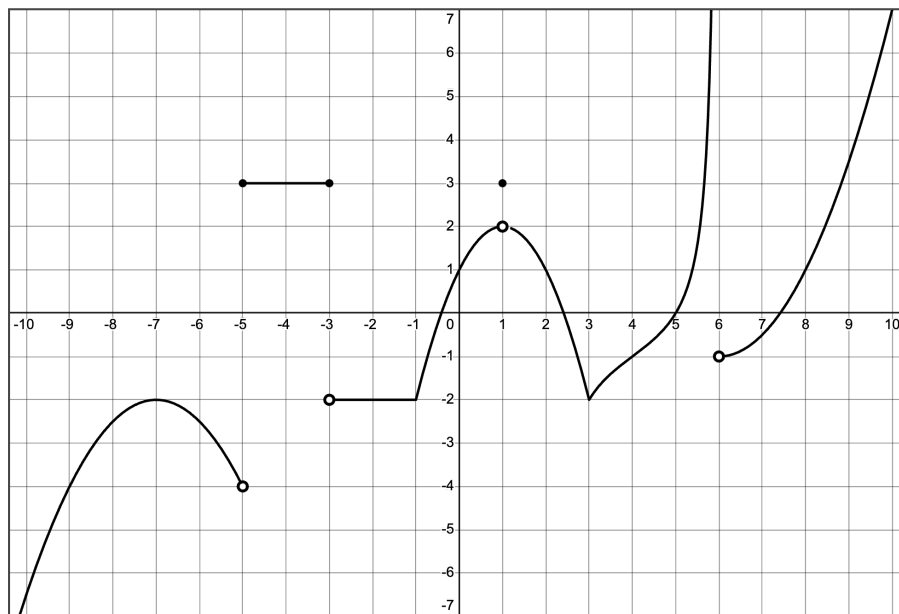
Consider the problem of finding the exact value of $\lim\limits_{x \to -5} \dfrac{x^3 + 5x^2 + 4x + 20}{x + 5}$. The method is to factor the numerator and cancel one of the factors. The factors of the numerator are $x + 5$ and $x^2 + 4$.

Write out the calculation in full, in correct notation which exhibits correctly the steps of the calculation. In particular, put "=" and "lim" everywhere they belong, and nowhere else. Start by writing $\lim\limits_{x \to -5} \dfrac{x^3 + 5x^2 + 4x + 20}{x + 5}$. Show at least the following steps:

- After factoring but before cancellation.

- After cancellation but before substituting $x = -5$.

- After substituting $x = -5$ but before possible simplification.

- The simplified final result, if the result in the previous step can be simplified.

There is no need to label the steps.

10. (6 points/part) For the function $y = L(x)$ graphed below, answer the following questions:

(a) Does $\lim_{x \to 1} L(x)$ exist? If so, what is it? If not, why not?

(b) Is L continuous at -1? Why or why not?

Extra credit. (5 extra credit points/part; grading will be harsher than on related problems on the main exam. Do not attempt this problem until you have done and checked your answer to all the ordinary problems on this exam. It will only be counted if you get a grade of B or better on the main part of this exam.)

Use the back of the page if needed.

(a) Let $f(x) = \sin(x)$. Find the 1167th derivative $f^{(1167)}(x)$.

(b) Let $g(x) = x^{1167}$. Find the 1166th derivative $g^{(1166)}(x)$.

(c) Let $f(x) = \cos(3x)$. Find the 1029th derivative $f^{(1029)}(x)$.