

SAMPLE MIDTERM 1, MATH 251 (PHILLIPS), SPRING 2025

CONTENTS

1. Midterm 1 Information	1
2. Sample Midterm 1	2
3. Extra Sample Problems for Midterm 1	3

1. MIDTERM 1 INFORMATION

At least 90% of the points on the real exam will be modifications of problems from the problems below, homework problems (*WeBWorK* and especially written homework), worksheets, and problems from the sample and real Midterms 0. Note, though, that the exact form of the functions to be differentiated and of the limits to be computed could vary substantially, and the methods required to do them might occur in different combinations. Word problems could have rather different descriptions, but similar methods will be used.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation for limits etc. in the book, in handouts, in files posted on the course website, and on the blackboard; *use it*. The right notation will help you get the mathematics right, and incorrect notation will lose points.

Here is the instruction sheet for Midterm 1:

- (1) DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.
- (2) The exam pages are **two sided**.
- (3) Closed book, except for a 3×5 file card, written on both sides.
- (4) The following are all prohibited: Calculators (of any kind), cell phones, laptops, iPods, electronic dictionaries, and any other electronic devices or communication devices. All electronic or communication devices you have with you must be turned completely off and put inside something (pack, purse, etc.) and out of sight.
- (5) The point values are as indicated in each problem; total 100 points.
- (6) Write all answers on the test paper. Use the back of the last page for long answers or scratch work. (If you do write an answer there, indicate on the page containing the problem where your answer is.)
- (7) Show your work. You must state what you did, legibly, clearly, correctly, and using correct notation. Among many other things, this means putting “ $=$ ”, limit symbols, etc. in all places where they belong, and not in any places where they don’t belong, as well as using all required parentheses. It also means organizing your work so that the order of the steps is clear, and it is clear how the steps are related to each other.
- (8) Correct answers with insufficient justification or accompanied by additional incorrect statements will not receive full credit. (Cross out any work you do not want to have

considered.) Correct guesses to problems requiring significant work, and correct answers obtained after a sequence of mostly incorrect steps, or for which the work is riddled with notation errors, will receive little or no credit.

- (9) Say what you mean. Credit will be based on what you say, not what you mean.
- (10) When exact values are specified, give answers such as $\frac{1}{7}$, $\sqrt{2}$, $\ln(23)$, or $\frac{2\pi}{9}$. Decimal approximations will not be accepted.
- (11) Final answers must always be simplified unless otherwise specified.
- (12) Grading complaints must be submitted in writing at the beginning of the class period after the one in which the exam is returned (usually by the Tuesday after the exam).
- (13) Time: 50 minutes.

2. SAMPLE MIDTERM 1

1. (a) (6 points) State carefully the definition of the derivative of a function.

(b) (13 points) If $f(x) = \frac{1}{8-x}$, compute the derivative $f'(2)$ *directly from the definition*.

(You can check your answer using a differentiation formula, but no credit will be given for just using the formula.)

(c) (1 point) Check your answer to Part (b) using a differentiation formula.

2. (10 points) Let g be a function such that $g'(x) = \sqrt[3]{3x^2 - 6} - 13$. Differentiate the function $f(t) = \sqrt{e} - t^2 g(t)$. (Your answer might involve the function g . You need not do this directly from the definition.)

3. (10 points) Differentiate the function $h(x) = \sin(6x^2 - 11x)$. (You need not do this directly from the definition.)

4. (20 points) A right circular cylinder is inscribed in a hemisphere (*not* a sphere) of radius $2\sqrt{3}$ inches. Find the largest possible volume of such a cylinder.

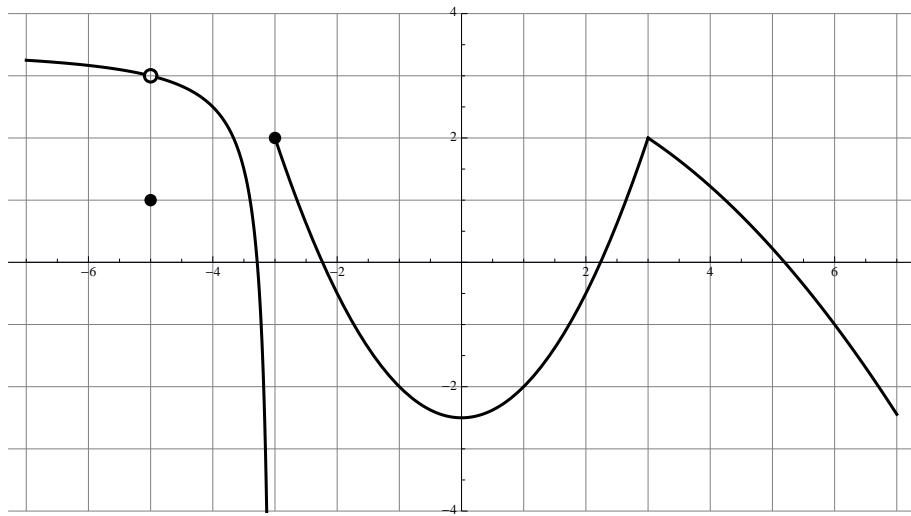
Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

5. (10 points) Find the exact value of the limit $\lim_{x \rightarrow 1} \frac{x-1}{x^2 - x - 6}$. Give reasons.

6. (10 points) Find the exact value of the limit $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x - 4}$. Give reasons.

7. (12 points) Use the methods of calculus to find the exact values of x at which the function $k(x) = 8x^3 - 36x^2 + 30x$ takes its absolute minimum and maximum on the interval $[0, 1]$.

8. For the function $y = k(x)$ graphed below, answer the following questions:



(a) (4 points.) Find $\lim_{x \rightarrow -5} k(x)$.

(b) (4 points.) Which of the following best describes $k'(4)$?

- $k'(4)$ does not exist.
- $k'(4)$ is close to 0.
- $k'(4)$ is positive and not close to 0.
- $k'(4)$ is negative and not close to 0.

3. EXTRA SAMPLE PROBLEMS FOR MIDTERM 1

There should probably be more differentiation problems.

9. (4 points/part) The following questions refer to the function whose graph is shown in problem 8.

(a) List all numbers a in $(-7, 7)$ such that h is not differentiable at a . Give reasons.

(b) List all numbers a in $(-7, 7)$ such that k is continuous at a but not differentiable at a . Give brief reasons.

(c) List all numbers a in $(-7, 7)$ such that k is differentiable at a but not continuous at a . Give brief reasons.

(d) Which of the following best describes $k'(7)$?

- $k'(7)$ does not exist.
- $k'(7)$ is close to 0.
- $k'(7)$ is positive and not close to 0.
- $k'(7)$ is negative and not close to 0.

(e) Find the largest interval containing 5 on which k is continuous.

10. (10 points) Find the exact value of the limit $\lim_{x \rightarrow 10} \frac{x - 10}{3(\sqrt{x} - \sqrt{10})}$. Give reasons.

11. (10 points) Find the exact value of the limit $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 + x - 12}$. Give reasons.

12. (12 points) Use the methods of calculus to find the exact values of x at which the function $f(x) = 3x^4 - 26x^3 + 45x^2 - 24$ takes its absolute minimum and maximum on the interval $[-1, 2]$.

Hint: $f'(x) = 6x(2x - 3)(x - 5)$. Also, the problem does not ask you to find the exact maximum and minimum values of h on the interval, only the exact values of x at which they occur.

13. (18 points) A lighthouse is located one mile offshore from a long straight beach. A town is located on the beach 3 miles from the point on the shore closest to the lighthouse. A cable is to go from the town to the lighthouse. It costs \$1000/mile on land and \$3000/mile underwater, and there is an extra expense of \$500 where it goes from the land to the water at the beach (but not at the lighthouse itself). You want to find the route and cost of the cheapest cable.

Set up, but **do not attempt to solve**, the appropriate maximization or minimization problem. That is, give a function $f(x)$, for x a suitable quantity related to the problem (say what x actually is!), give a suitable domain, and say whether you want to maximize or minimize f in this domain. Provide justification for all steps (possibly including a picture).

14. (10 points/part) Differentiate the following functions:

(a) $f(y) = (7y^3 + \frac{1}{2})(\frac{1}{8}y^7 - 16\sqrt{y} + \frac{2}{3})$.

(b) $h(t) = \frac{\sqrt[3]{t} - 2\pi}{\sqrt[3]{t} + 2\pi}$.

(c) Given that $h'(x) = 3h(x)$, find $\frac{d}{dx} \left(\frac{x}{h(x)} \right)$. (Your answer might involve the function h .)

(d) $f(x) = \sin((x^2 - k)^{17})$, where k is a constant.

15. Let f and g be functions such that:

$$f(-3) = -5, \quad f'(-3) = 12, \quad g(-3) = 2, \quad \text{and} \quad g'(-3) = -3$$

and

$$f(2) = 7, \quad f'(2) = 3, \quad g(2) = -3, \quad \text{and} \quad g'(2) = 2.$$

Let $h(x) = f(g(x))$.

(a) (2 points) Find $h(2)$. (You will not need to use all the information provided.)

(b) (8 points) Find $h'(2)$. (You will not need to use all the information provided.)