MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 4 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Suppose $g(x) = 2x^3 - 10x^2 + 2$. Find the exact value of $g(3)$.

Solution:

$$g(3) = 2(3)^3 - 10(3)^2 + 2 = 2(27) - 10(9) + 2 = 54 - 90 + 2 = -34.$$

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2. Write as a single fraction, and simplify as much as possible: $\dfrac{1}{x+3} - \dfrac{1}{x-4}$

Solution:

$$\frac{1}{x+3} - \frac{1}{x-4} = \frac{x-4}{(x+3)(x-4)} - \frac{x+3}{(x+3)(x-4)} = \frac{x-4-x-3}{(x+3)(x-4)} = -\frac{7}{(x+3)(x-4)}.$$

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3. Find all real solutions to the equation $\dfrac{e^x + x^3}{x^3} = 1$. If no real solution exists, write "no solution".

Solution:

$$\frac{e^x + x^2}{x^2} = 1$$
$$e^x + x^2 = x^2$$
$$e^x = 0.$$

Since e^x can never be zero, there are no solutions.

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4. Find all real numbers a such that $\frac{1}{|a|} = -\frac{1}{a}$.

Solution: Clearly we can't have $a = 0$. For $a \neq 0$, by taking reciprocals, we see that the equation is equivalent to $|a| = -a$. This is true if and only if $a \leq 0$. Since we must exclude $a = 0$ in the original equation, it follows that $\frac{1}{|a|} = -\frac{1}{a}$ if and only if $a < 0$.

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5. Find all real solutions to the equation $x(x-1) = 20$. If no real solution exists, write "no solution".

Solution:

$$x(x-1) = 20$$
$$x^2 - x = 20$$
$$x^2 - x - 20 = 0$$
$$(x+4)(x-5) = 0$$
$$x = -4 \qquad \text{or} \qquad x = 5.$$

Since there is no partial credit, no credit is given for only one of the two solutions.

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6. Let $f(x) = 2 - x$. Evaluate the expression $f(x+3) - f(2x+1)$, and simplify it as much as possible.

Solution:

$$f(x+3) - f(2x+1) = 2 - (x+3) - (2 - (2x+1)) = 2 - x - 3 - 2 + 2x + 1 = x - 2.$$

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7. Multiply out: $(6x-2)(3x+4)$.

Solution:

$$(6x-2)(3x+4) = 18x^2 + 24x - 6x - 8 = 18x^2 + 18x - 8.$$

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8. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{2t^2+2}{2t^2+6}$

Solution:

$$\frac{2t^2+2}{2t^2+6} = \frac{2(t^2+1)}{2(t^2+3)} = \frac{t^2+1}{t^2+3}.$$

The last expression can't be further simplified.

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9. Assuming $y > 0$, write the expression $\dfrac{\sqrt[7]{y}}{7}$ as a numerical constant (possibly a fraction) multiplied by a power of y. (y may not appear in a denominator.)

Solution:

$$\frac{\sqrt[7]{y}}{7} = \frac{1}{7}y^{1/7}.$$

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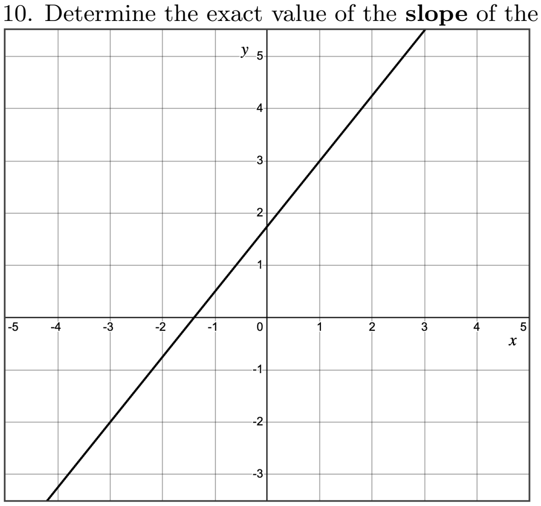
10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (1, 3)$ are on the line. Therefore the slope is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{3-(-3)} = \frac{5}{4}.$$

Another approach is to simply observe from the graph that the line goes up 5 units for each 4 units to the right.
