MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 3 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Find all real solutions to the equation $4x^{-2} = 1$. If no real solution exists, write "no solution".

Solution:

$$4x^{-2} = 1$$
$$4 = x^2$$
$$x = \pm\sqrt{4} = \pm 2$$

Since there is no partial credit, no credit is given for only one of the two solutions.

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2. Find all real numbers a such that $(2a, -a)$ is in the second quadrant (and not on any of the coordinate axes).

Solution: The point $(2a, -a)$ is in the second quadrant if and only if both $2a < 0$ and $-a > 0$, which happens if and only if $a < 0$.

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3. Suppose $q(x) = 2x^4 + 4x^2 - x$. Find the exact value of $q(10)$.

Solution:

$$q(10) = 2(10)^4 + 4(10)^2 - 10 = 2 \cdot 10{,}000 + 4 \cdot 100 - 10 = 20{,}000 + 400 - 10 = 20{,}390.$$

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4. Let $f(x) = 3 - x$. Evaluate the expression $f(1 - x) - f(1)$, and simplify it as much as possible.

Solution:

$$f(1 - x) - f(1) = 3 - (1 - x) - (3 - 1) = 3 - 1 + x - 3 + 1 = x.$$

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5. Write as a single fraction, and simplify as much as possible: $\dfrac{6}{c-4} - \dfrac{1}{c-2}$

Solution:

$$\frac{6}{c-4} - \frac{1}{c-2} = \frac{6(c-2)}{(c-4)(c-2)} - \frac{c-4}{(c-4)(c-2)} = \frac{6c - 12 - c + 4}{(c-4)(c-2)} = \frac{5c-8}{(c-4)(c-2)}.$$

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6. Find all real solutions to the equation $\dfrac{16w^{-1}}{w+6} = 1$. If no real solution exists, write "no solution".

Solution:

$$\frac{16w^{-1}}{w+6} = 1$$

Multiply by $w+6$:

$$16w^{-1} = w+6$$

Multiply by w:

$$16 = w^2 + 6w$$

$$w^2 + 6w - 16 = 0$$

$$(w-2)(w+8) = 0$$

$$w = 2 \qquad \text{or} \qquad w = -8.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by $w+6$ at the first step, and multiplying both sides by w at the second step, did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

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7. Multiply out: $(7y-2)(-4y+3)$.

Solution:

$$(7y-2)(-4y+3) = -28y^2 + 21y + 8y - 6 = -28y^2 + 29y - 6.$$

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8. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{3\cos(7x)+12}{3\cos(7x)+6}$

Solution:

$$\frac{3\cos(7x)+12}{3\cos(7x)+6} = \frac{3(\cos(7x)+4)}{3(\cos(7x)+2)} = \frac{\cos(7x)+4}{\cos(7x)+2}.$$

The last expression can't be further simplified.

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9. Assuming $t > 0$, write the expression $\dfrac{\sqrt{t}}{2t}$ as a numerical constant (possibly a fraction) multiplied by a power of t. (t may not appear in a denominator.)

Solution:

$$\frac{\sqrt{t}}{2t} = \frac{t^{1/2}}{2t} = \frac{1}{2}t^{\frac{1}{2}-1} = \frac{1}{2}t^{-1/2}.$$

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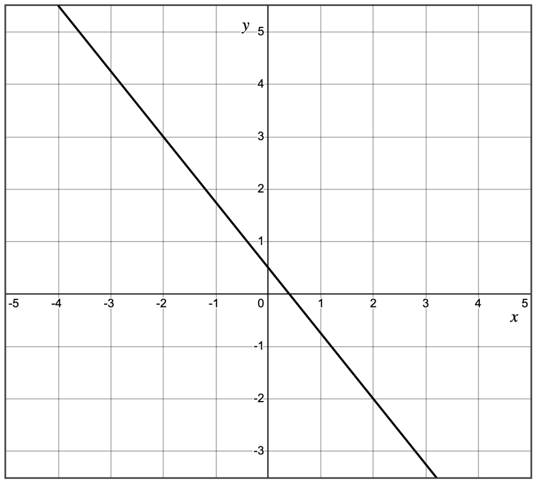
10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (2, -2)$ and $(x_2, y_2) = (-2, 3)$ are on the line. Therefore the slope is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{-2 - 2} = \frac{5}{-4} = -\frac{5}{4}.$$

Another approach is to simply observe from the graph that the line goes down five units for each four units to the right.
