MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 2 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Find all real solutions to the equation $\frac{6}{x} + \frac{7}{x^2} = 1$. If no real solution exists, write "no solution".

Solution: This is a quadratic equation in $1/x$:

$$\frac{6}{x} + \frac{7}{x^2} = 1$$

$$7\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) - 1 = 0$$

$$\left(7\left(\frac{1}{x}\right) - 1\right)\left(\frac{1}{x} + 1\right) = 0$$

$$\frac{1}{x} = \frac{1}{7} \qquad \text{or} \qquad \frac{1}{x} = -1.$$

$$x = -1 \qquad \text{or} \qquad x = 7.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x at the last step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

Alternate solution: Multiply through by x^2 first, getting:

$$6x + 7 = x^2$$

$$x^2 - 6x - 7 = 0$$

$$(x+1)(x-7) = 0$$

$$x = -1 \qquad \text{or} \qquad x = 7.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x^2 at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

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2. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\frac{x^3 + 7x}{x^3 + 2x}$

Solution:

$$\frac{x^3 + 7x}{x^3 + 2x} = \frac{x(x^2 + 7)}{x(x^2 + 2)} = \frac{x^2 + 7}{x^2 + 2}.$$

The last expression can't be further simplified.

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3. Assuming $x > 0$, write the expression $\frac{x^2}{2\sqrt{x}}$ as a numerical constant (possibly a fraction) multiplied by a power of x. (x may not appear in a denominator.)

Solution:

$$\frac{x^2}{2\sqrt{x}} = \frac{x^2}{2x^{1/2}} = \frac{1}{2}x^{2-\frac{1}{2}} = \frac{1}{2}x^{3/2}.$$

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4. Multiply out: $(y-5)\big(y^2 + 3y - 2\big)$.

Solution:

$$(y-5)\big(y^2 + 3y - 2\big) = y^3 - 5y^2 + 3y^2 - 15y - 2y + 10 = y^3 - 2y^2 - 17y + 10.$$

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5. Let $f(x) = 7 - x$. Evaluate the expression $f(2-x) - f(x)$, and simplify it as much as possible.

Solution:

$$f(2-x) - f(x) = 7 - (2-x) - (7-x) = 7 - 2 + x - 7 + x = 2x - 2.$$

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6. Find all real numbers a such that $|a+2| = -a-2$.

Solution: $|a+2| = -a-2$ if and only if $|a+2| = -(a+2)$. Since $|x| = -x$ if and only if $x \leq 0$, this happens if and only if $a+2 \leq 0$, which is true if and only if $a \leq -2$.

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7. Find all real solutions to the equation $3y^{-3} = 0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by y^3 to get $3 = 0$. Therefore there are no solutions. (Alternatively, write $3y^{-3} = 3/y^3$, which can obviously never be zero.)

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8. Suppose $f(x) = 2x^3 - 4x^2 - x$. Find the exact value of $f(-10)$.

Solution:

$$f(-10) = 2(-10)^3 - 4(-10)^2 - (-10) = 2(-1000) - 4(100) + 10 = -2000 - 400 + 10 = -2390.$$

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9. Write as a single fraction, and simplify as much as possible: $\dfrac{2}{p+4} - \dfrac{1}{p+5}$

Solution:

$$\frac{2}{p+4} - \frac{1}{p+5} = \frac{2(p+5)-(p+4)}{(p+4)(p+5)} = \frac{2p+10-p-4}{(p+4)(p+5)} = \frac{p+6}{(p+4)(p+5)}.$$

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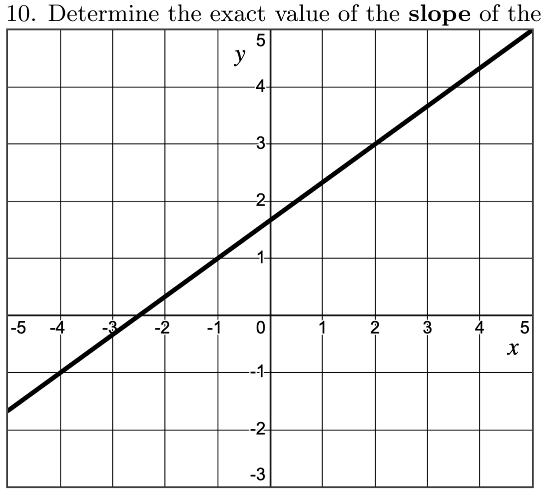
10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (-1, 1)$ are on the line. Therefore the slope is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{3-1}{2-(-1)} = \frac{2}{3}.$$

Another approach is to simply observe from the graph that the line goes up two units for each three units to the right.