MATH 251 (PHILLIPS) MIDTERM 0 EXTRA PROBLEM LIST SET 1 SOLUTIONS

Warning: Not enough proofreading has been done! (People have gotten extra credit for catching previous errors.)

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1. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{\sin(7x)+7}{\sin(7x)-7}$

Solution: The expression $\dfrac{\sin(7x)+7}{\sin(7x)-7}$ can't be simplified.

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2. Assuming $c > 0$, write the expression $\dfrac{2}{3\sqrt[3]{c}}$ as a numerical constant (possibly a fraction) multiplied by a power of c. (c may not appear in a denominator.)

Solution:

$$\frac{2}{3\sqrt[3]{c}} = \frac{2}{3c^{1/3}} = \frac{2}{3} \cdot c^{-1/3}.$$

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3. Find all real solutions to the equation $\dfrac{e^{-5x}}{x^2} = 0$. If no real solution exists, write "no solution".

Solution: Multiply both sides by x^2 to get $e^{-5x} = 0$. Since e^{-5x} can never be zerothere are no solutions.

(Alternatively, since e^{-5x} can never be zero, it is obvious that $\frac{e^{-5x}}{x^2}$ can never be zero.)

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4. Let $g(x) = 7 - 4x$. Evaluate the expression $\dfrac{g(5+h)-g(5)}{h}$, and simplify it as much as possible.

Solution:

$$\frac{g(5+h)-g(5)}{h} = \frac{7-4(5+h)-(7-4\cdot 5)}{h} = \frac{7-20-4h-7+20}{h} = \frac{-4h}{h} = -4.$$

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5. Write as a single fraction, and simplify as much as possible: $\dfrac{3}{x+4} - \dfrac{1}{x-5}$

Solution:

$$\frac{3}{x+4} - \frac{1}{x-5} = \frac{3(x-5)-(x+4)}{(x+4)(x-5)} = \frac{3x-15-x-4}{(x+4)(x-5)} = \frac{x-19}{(x+4)(x-5)}.$$

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6. Find all real solutions to the equation $\dfrac{8}{x} - x = -2$. If no real solution exists, write "no solution".

Solution:

$$\begin{aligned} \frac{8}{x} - x &= -2 \\ 8 - x^2 &= -2x \\ x^2 - 8 &= 2x \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \end{aligned}$$

$$x = 4 \qquad \text{or} \qquad x = -2.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by x at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

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7. Suppose $f(x) = 3x^3 + 4x^2 - 2x$. Find the exact value of $f(-2)$.

Solution:

$$f(-2) = 3(-2)^3 + 4(-2)^2 - 2(-2) = 3(-8) + 4(4) + 4 = -24 + 16 + 4 = -4.$$

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8. Find the domain of the function $r(x) = (-x)^{-1/4}$.

Solution: Rewrite $r(x) = \dfrac{1}{\sqrt[4]{-x}}$. We can now see that x is in the domain if and only if $-x \geq 0$ (so that $\sqrt[4]{-x}$ is defined) and $\sqrt[4]{-x} \neq 0$ (so that $\dfrac{1}{\sqrt[4]{-x}}$ is also defined). The first condition requires $x \leq 0$, and the second rules out $x = 0$, so the answer is all real x with $x < 0$.

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9. Multiply out: $(3t - 2)(4t - 6)$.

Solution:

$$(3t - 2)(4t - 6) = 12t^2 - 18t - 8t + 12 = 12t^2 - 26t + 12.$$

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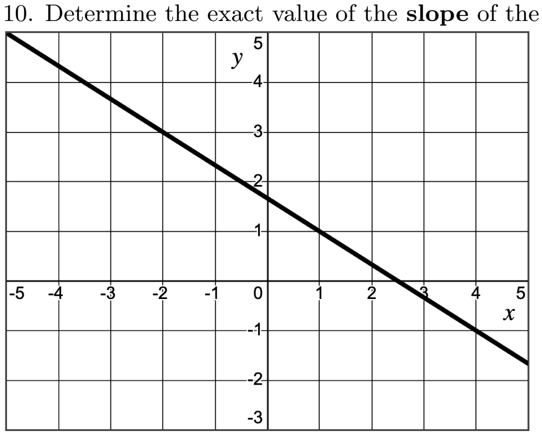
10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (-2, 3)$ are on the line. Therefore the slope is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - 3}{1 - (-2)} = \frac{-2}{3} = -\frac{2}{3}.$$

Another approach is to simply observe from the graph that the line goes down two units for each three units to the right.
