REASONS FOR INCLUSION OF PROBLEMS ON MIDTERM 0

This file describes types of problems that are used on Midterm 0, and why they are there.

1. Subtracting fractions

Sample problem: Write as a single fraction, and simplify as much as possible: $\dfrac{2}{x-1} - \dfrac{1}{x+2}$.

Subtraction of fractions is needed in various places, for example, calculating a difference quotient for the function $f(x) = \dfrac{1}{x+2}$ and the limits at $\pm\infty$ of a difference of fractions.

This problem also tests correct distribution of minus signs across parentheses, a source of costly errors in many problem in the course.

2. Rational Function Simplification

Sample problems: Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{6x^2+3}{6x^2+12}$.

Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{\sin(7x)+7}{\sin(7x)-7}$.

This problem tests for improper cancellation when simplifying rational expressions and similar fractions. Improper cancellation happens in many places, often with disastrous results, for example when finding limits of fractions which are supposed to be found by cancelling common factors or using L'Hospital's Rule, when simplifying the result of implicit differentiation, when simplifying the result of using the quotient rule, and when trying to solve $f'(x) = 0$ for functions arising in many applied optimization problems.

3. Rewriting roots as exponents

Sample problem: Assuming $x > 0$, write the expression $\dfrac{5}{\sqrt[5]{x}}$ as a numerical constant (possibly a fraction) multiplied by a power of x.

This operation is needed when trying to differentiate expressions like $\dfrac{5}{\sqrt[5]{x}}$ or $\sqrt{x^2+2x+8}$. Besides general differentiation problems and applications of differentiation to the shape of a graph, expressions like $\sqrt{x^2+9}$ appear in some kinds of related rates and applied optimization problems.

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4. Recognition that $1/x^2 = 0$ and similar equations have no solutions

Sample problem: Find all real solutions to the equation $8\left(2 - \frac{1}{x^3}\right) = 16$. If no real solution exists, write "no solution".

Equations in which a fraction with a power of x in the denominator, or involving a negative power of x, must equal zero occur in several common types of applied optimization problems, and in other cases of solving the equation $f'(x) = 0$. For such equations, $x = 0$ is **not** a solution.

5. Polynomial evaluation

Sample problem: Suppose $g(x) = 2x^3 + 4x^2 - x$. Find the exact value of $g(-10)$.

Several kinds of mistakes in such computations occur much more often than they should. Calculations like this occur in multiple places, one of the most common being finding absolute minimums and maximums of polynomial functions on closed intervals.

6. Function Evaluation

Sample problem: Let $h(x) = 2 - x^2$. Evaluate the expression $h(2) - h(x + 2)$, and simplify it as much as possible.

Calculations of this general nature occur when finding derivatives from the definition, and sometimes elsewhere.

This problem also tests correct distribution of minus signs across parentheses, a source of costly errors in many problem in the course.

7. Recognition that $-x$ can be positive

Sample problem: Find all real numbers a such that $|a| = -a$.

Recognizing that $-x$ can be positive is important in a lecture discussing symmetry of functions, such as $\sin(x)$ and $\cos(x)$, and when differentiating $f(x) = |x|$.

It is also related to the fact that if an object is falling at -6 meters per second, it is rising at 6 meters per second.

This mistake is aggravated by the common bad practice in speech of reading $-x$ as "~~negative x~~" instead of the correct "minus x". It is also aggravated by the common practice in physics and chemistry of putting in explicit minus signs to make all proportionality constants positive.

8. Solving Quadratic Equations in Nonstandard Form

Sample problems: Find all real solutions to the equation $\frac{4}{x^2 + 3x} = 1$. If no real solution exists, write "no solution".

Find all real solutions to the equation $8 - x^2 = 2x$. If no real solution exists, write "no solution".

Find all real solutions to the equation $x(x - 1) = 20$. If no real solution exists, write "no solution".

Find all real solutions to the equation $x - 3 = 4x^{-1}$. If no real solution exists, write "no solution".

The equation $f'(x) = 0$ is sometimes a quadratic (or cubic, etc.) equation in disguise. For example, optimization word problems often involve functions of the form $f(x) = ax + bx^{-1}$.

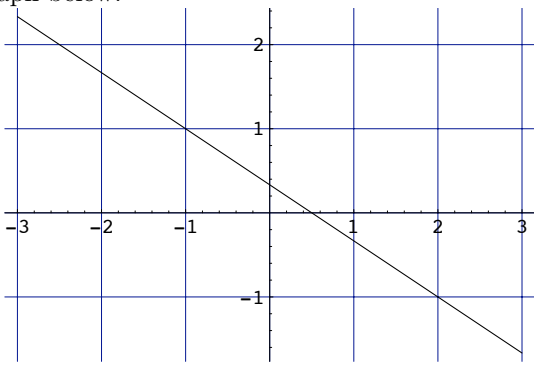
9. Polynomial Multiplication

Sample problems: Multiply out: $(x + 2)(x^2 - x + 1)$.
Multiply out: $(7x + 2)(5x - 4)$.

This sort of thing appears in scattered places throughout the course, and mistakes are too common.

10. Slopes of Lines from Graphs

Sample problems: Determine the exact value of the **slope** of the line in the graph below.



It is essential in this course to be able to at least estimate slopes of tangent lines from their graphs.