MATH 251 (PHILLIPS) MIDTERM 0 VERSION 2, F 11 April 2025

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INSTRUCTIONS: No books, notes, or calculators are permitted on this test. All answers must be simplified as much as possible. Write all answers in the spaces provided at the right. Do scratchwork on the back or on blank paper provided for this purpose. *No partial credit.* Time: 30 minutes.

1. Suppose $p(x) = 3x^3 - 5x^2 + 2$. Find the exact value of $p(2)$.

Solution:

$$p(2) = 3(2)^3 - 5(2)^2 + 2 = 3(8) - 5(4) + 2 = 24 - 20 + 2 = 6.$$

2. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{z^3 + 3z}{z^3 + 6z}$

Solution:

$$\frac{z^3 + 3z}{z^3 + 6z} = \frac{z(z^2 + 3)}{z(z^2 + 6)} = \frac{z^2 + 3}{z^2 + 6}.$$

The last expression can't be further simplified.

3. Find all real solutions to the equation $1 = 10z^{-2} - 3z^{-1}$. If no real solution exists, write "no solution".

Solution:

$$1 = 10z^{-2} - 3z^{-1}$$

Multiply by z^2:

$$z^2 = 10 - 3z$$

$$z^2 + 3z - 10 = 0$$

$$(z - 2)(z + 5) = 0$$

$$z = 2 \qquad \text{or} \qquad z = -5.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by z^2 at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

4. Find all real solutions to the equation $3(4 - y^{-3}) = 12$. If no real solution exists, write "no solution".

Solution: Expanding the left hand side, we get $12 - 3y^{-3} = 12$, or $-3y^{-3} = 0$. Multiply both sides by y^3 to get $-3 = 0$. Thus, there is no solution. (Alternatively, write $-3y^{-3} = 3/y^3$, which can obviously never be zero.)

5. Multiply out: $(5x - 3)(2x + 4)$.

Solution:

$$(5x - 3)(2x + 4) = 5 \cdot 2 \cdot x^2 + 5 \cdot 4 \cdot x - 3 \cdot 2 \cdot x - 3 \cdot 4 = 10x^2 + 14x - 12.$$

6. Write as a single fraction, and simplify as much as possible: $\dfrac{2}{z - 1} - \dfrac{1}{z - 7}$

Solution:

$$\frac{2}{z-1}-\frac{1}{z-7}=\frac{2(z-7)}{(z-1)(z-7)}-\frac{z-1}{(z-1)(z-7)}=\frac{2z-14-(z-1)}{(z-1)(z-7)}=\frac{z-13}{(z-1)(z-7)}.$$

7. Assuming $y>0$, write the expression $\dfrac{2\sqrt[7]{y}}{7y}$ as a numerical constant (possibly a fraction) multiplied by a power of y. (y may not appear in a denominator.)

Solution:

$$\frac{2\sqrt[7]{y}}{7y}=\frac{2y^{1/7}}{7y}=\frac{2}{7}y^{\frac{1}{7}-1}=\frac{2}{7}y^{-\frac{6}{7}}.$$

8. Find the domain of the function $f(x)=\sqrt{-x}$.

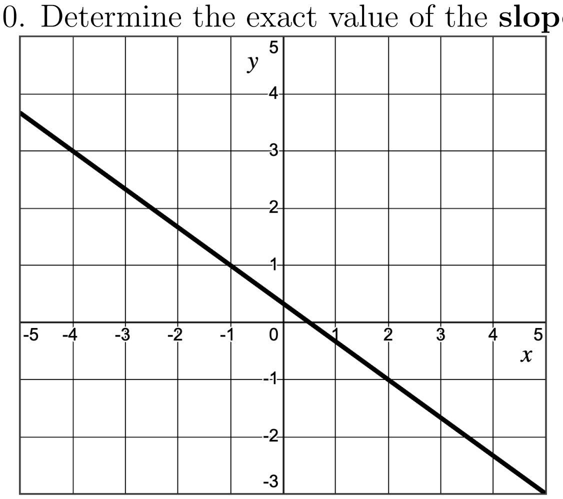
Solution: $\sqrt{-x}$ is defined if and only if $-x\geq 0$, which happens if and only if $x\leq 0$. Therefore the domain is $(-\infty, 0]$.

9. Let $f(x)=7-x$. Evaluate the expression $f(17)-f(2x-3)$, and simplify it as much as possible.

Solution:

$$f(17)-f(2x-3)=7-17-\big(7-(2x-3)\big)=7-17-7+2x-3=2x-20.$$

10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1,y_1)=(-1,1)$ and $(x_2,y_2)=(2,-1)$ are on the line. Therefore the slope is

$$\frac{y_1-y_2}{x_1-x_2}=\frac{1-(-1)}{-1-2}=\frac{2}{-3}=-\frac{2}{3}.$$

Another approach is to simply observe from the graph that the line goes down two units for each three units to the right.