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MATH 251 (PHILLIPS) MIDTERM ZERO (SAMPLE 2)

Turn in this version of the sample Midterm Zero as homework Tuesday 1 April 2025.

(1) **Unlike** the real version, show work in the space below the problem. It will be graded for partial credit.

(2) **Work must use fully correct notation, and correctly show what your steps were.** It must have "=" where it is supposed to be, and not where it is not supposed to be. See Section 6 of the online notation sheet. Also see the specific notation warnings on some problems here and on the other sample; these will **not** appear on the real Midterm Zero.

(3) All answers must be simplified as much as possible.

The real Midterm Zero will allow no books, notes, calculators, or other electronic devices, and will have no partial credit.

1. Suppose $f(x) = 2x^3 - 3x^2 - 2x$. Find the exact value of $f(3)$.

Solution:

$$f(3) = 2(3)^3 - 3(3)^2 - 2(3) = 2(27) - 3(9) - 6 = 54 - 27 - 6 = 21.$$

2. Write as a single fraction, and simplify as much as possible: $\dfrac{1}{x+3} - \dfrac{1}{x-7}$

(See the reminder on fraction notation on the other sample, and Section 3 of the online notation sheet.)

Solution:

$$\frac{1}{x+3} - \frac{1}{x-7} = \frac{x-7}{(x+3)(x-7)} - \frac{x+3}{(x+3)(x-7)} = \frac{x-7-x-3}{(x+3)(x-7)} = -\frac{10}{(x+3)(x-7)}.$$

3. Let $f(x) = 1 - x$. Evaluate the expression $f(2x-5) - f(x+3)$, and simplify it as much as possible.

Solution:

$$f(2x-5) - f(x+3) = 1 - (2x-5) - (1 - (x+3)) = 1 - 2x + 5 - 1 + (x+3) = -x + 8.$$

4. Find all real numbers b such that $(-7, -b)$ is in the second quadrant (and not on any of the coordinate axes). (Notation reminder: Be sure to use the right variable!)

Solution: $(-7, -b)$ is in the second quadrant if and only if $-b > 0$, which happens if and only if $b < 0$.

5. Find all real solutions to the equation $\dfrac{12}{z^2 + 4z} = 1$. If no real solution exists, write "no solution". (Notation reminder: Be sure to use the right variable!)

Solution:

$$\frac{12}{z^2 + 4z} = 1$$

Multiply by $z^2 + 4z$:

$$12 = z^2 + 4z$$

$$z^2 + 4z - 12 = 0$$

$$(z-2)(z+6) = 0$$

$$z = 2 \qquad \text{or} \qquad z = -6.$$

Note that both answers actually are solutions to the original equation, that is, that multiplying both sides by $z^2 + 4z$ at the first step did not introduce any extraneous solutions.

Since there is no partial credit, no credit is given for only one of the two solutions.

6. Multiply out: $(3y - 2)(-7y + 5)$.

Solution:

$$(3y - 2)(-7y + 5) = -21y^2 + 14y + 15y - 10 = -21y^2 + 29y - 10.$$

7. Simplify the following expression as much as possible. If no simplification is possible, write "not possible": $\dfrac{2w^2 + 6}{w^2 + 6}$

Solution: The expression $\dfrac{2w^2 + 6}{w^2 + 6}$ can't be simplified.

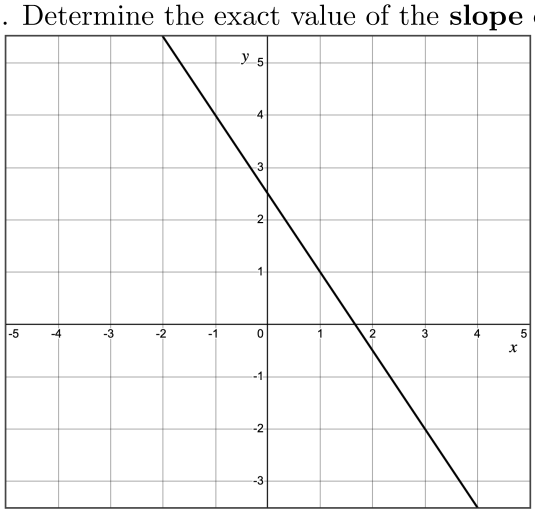
8. Assuming $t > 0$, write the expression $\dfrac{1}{6\sqrt[6]{t}}$ as a numerical constant (possibly a fraction) multiplied by a power of t.

Solution: $\dfrac{1}{6\sqrt[6]{t}} = \dfrac{1}{6t^{1/6}} = \dfrac{1}{6} \cdot \dfrac{1}{t^{1/6}} = \dfrac{1}{6}t^{-1/6}$.

9. Find all real solutions to the equation $4\left(\dfrac{1}{x^2} + 3\right) = 12$. If no real solution exists, write "no solution".

Solution: The equation implies $\dfrac{1}{x^2} + 3 = 3$, so $\dfrac{1}{x^2} = 0$. Multiply both sides by x^2 to get $1 = 0$. Therefore there are no solutions. (Alternatively, it is obvious that $\dfrac{1}{x^2}$ can never be zero.)

10. Determine the exact value of the **slope** of the line in the graph below.



Solution: You can tell by reading the graph that the points $(x_1, y_1) = (3, -2)$ and $(x_2, y_2) = (1, 1)$ are on the line. Therefore the slope is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 1}{3 - 1} = -\frac{3}{2}.$$

Another approach is to simply observe from the graph that the line goes down three units for each two units to the right.

The formula ~~$y = -\frac{3}{2}x + \frac{5}{2}$~~ is wrong: it is the equation of the line, not its slope. The answer ~~$1\frac{1}{2}$~~ won't be accepted: see the notation page about not using mixed fractions.