MATHEMATICS 251 LEARNING OBJECTIVES: SPRING QUARTER, 2025

The most important single learning objective is that a successful student in this course **should be able to model and solve a wide class of optimization and other modelling problems that are accessible to differential calculus.** Much of the other material covered in this course is necessary for that objective. So subgoals include:

(1) A successful student should be able to differentiate. This is necessary to use calculus to solve optimization problems (or any other kind of problem—see the additional objectives below).
(2) A successful student should be able to sketch graphs of functions. This is necessary to help identify where to search for local and global extremums when trying to optimize.
(3) A successful student should understand some basic facts about limits. This is needed for two reasons: to incorporate an understanding of the geometric interpretation of the derivative as the slope of the tangent line of a graph, and also to aid in sketching graphs of functions exhibiting asymptotic or discontinuous behavior.
(4) A successful student should be able to solve related rates problems.
(5) A successful student should be able to find the linear approximation to a function at a specific value of the variable, graph the linear approximation and the function on the same pair of axes, and use the linear approximation to find approximations to values of the function near the point at which the approximation is taken.

In more detail, a successful student should be able to the following. In all cases, a successful student should be able to write the solution in a well organized manner, showing a sufficient number of steps, written using fully correct notation and terminology, and correctly showing what the steps are.

(1) Evaluate limits using the algebraic limit laws.
(2) Identify limits at $\pm\infty$ for rational functions and similar functions.
(3) Identify limits of rational functions involving cancellation of linear factors from numerator and denominator.

(4) Compute left and right limits for a function (or decide they do not exist), given an expression for the function.
(5) Identify the points where common functions are continuous or differentiable, and the same for functions given graphically.
(6) Identify the points where graphically given functions are continuous or differentiable.
(7) Identify limits, as well as left and right limits, for functions given graphically.
(8) State and use the product rule, quotient rule, chain rule, and linearity rules for derivatives.
(9) State the definition of the derivative in terms of a limit of difference quotients.
(10) Interpret, including units, the derivative as an instantaneous rate of change of a quantity defined in an applied context.
(11) Recognize the derivative as the slope of the tangent line.
(12) Use calculus to approximate the value of a function near a point p, given information about the function and its derivatives at p.
(13) Compute derivatives of functions involving polynomials, exponentials, logarithms, trigonometric functions, and inverse trigonometric functions, using a combination of theorems, differentiation rules, and definitions.
(14) Find the equation for the tangent line of a curve at a given point.
(15) Calculate derivatives using implicit differentiation
(16) Use the methods of calculus to find asymptotes, local minimums and maximums, intervals of concavity, intervals where the function is increasing or decreasing, and inflection points. Relate these properties to the graph of the function.
(17) Find extremums of a function on open and closed intervals.
(18) Solve optimization problems, including word problems.
(19) Solve related rates problems, including word problems.
(20) Use L'Hopital's rule to evaluate indeterminate forms of limits, including cases requiring multiple applications.
(21) Use the Intermediate Value Theorem to prove that roots of a function exist in a given closed interval.
(22) Use Newton's method to approximate solutions to equations that they cannot solve explicitly.
(23) Correctly use the notation and terminology of the course. Correct use of terms and symbols is taken as evidence of understanding of their meaning. In addition, correct use of terms

and symbols is like using correct grammar and spelling in an essay or term paper. Here is an incomplete list of examples:

(a) Using correct notation for derivatives.
(b) Putting the symbol $\lim_{x \to a}$ in places where it belongs, and not putting this symbol in places where it doesn't belong.
(c) Putting the symbol $=$ in places where it belongs, and not in places where it doesn't belong. (This course provides new contexts in which this is important.)
(d) Using parentheses when needed. (This course provides new contexts in which this is important.)
(e) Recognizing that expressions like $\frac{\infty}{\infty}$, $\frac{0}{0}$, $\frac{0}{\infty}$, $0 \cdot \infty$, $17 \cdot \infty$, $\frac{17}{\infty}$, etc. are not numbers and therefore may not appear in equations.