SAMPLE FINAL EXAM QUESTIONS, MATH 251 (PHILLIPS), SPRING 2025

Contents

1. Final Exam Information

At least 90% of the points on the real exam will be modifications of problems from Midterms 1 and 2 from the last time I taught the course, real and sample midterms and quizzes so far in this course (including Midterms 0), the problems below, homework problems (including written homework and WeBWorK), and worksheet problems. Note, though, that the exact form of the functions to be differentiated and of the limits to be computed could vary substantially, and the methods required to do them might occur in different combinations. Word problems could have rather different descriptions, but similar methods will be used.

Be sure to get the notation right! (This is a frequent source of errors.) You have seen the correct notation for limits etc. in the book, in handouts, in files posted on the course website, and on the blackboard; *use it*. The right notation will help you get the mathematics right, and incorrect notation will lose points.

The exam will be 200 points, 2 hours, and will be (in my estimation; possibly wrong) at least a little less than twice the length of a midterm. The section "Sample Final Exam" has been hastily assembled from old exams, without careful consideration of length.

I will allow one page of notes, rather than just a file card.

Exams will be available for inspection when they are graded, probably by Thursday 20 March. I keep the originals, but you can get a copy on request. Grading complaints must be submitted in writing before final grades are turned in. Extra credit for catching errors is available through 5:00 pm Thursday 20 March, possibly later (but no promises).

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2. Sample Final Exam

The problems here are intended to give a reasonable idea of how much of the final exam will come from each part (broadly interpreted) of the course. Finer details may well be different.

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1. (11 points/part.) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist or there is not enough information to evaluate them. Give justification in all cases (not just heuristic arguments). Remember to use correct notation.

(a) $\lim_{x\to 3} \dfrac{x^2-9}{\cos(x)-3}$.

(b) $\lim_{y\to\infty} \dfrac{2y+216}{13y-5\sin(y)}$. (Be sure to show your work!)

(c) $\lim_{x\to 0} \dfrac{5x^2}{1-\cos(7x)}$.

(d) $\lim_{x\to 4^+} \dfrac{e^{-3x}}{x-4}$. (Be sure to show your work!)

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2. (6 points.) Let $w(t)$ be the water flow at time t in a river at a particular measuring station. Assume that t is measured in days, and that $w(t)$ is measured in m^3/sec. What are the units of $w'(t)$?

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3. (12 points) Find the equation of tangent line to the graph of $g(x) = 2x + 4\sqrt{3x-2}$ at $x = 2$. You need not calculate the derivative directly from the definition.

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4. (10 points/part) Differentiate the functions as requested.

(a) Find $f'(x)$, where $f(x) = \pi^3 + \dfrac{2x+1}{x^2+1}$.

(b) Let $f(t) = e^{7t+\arcsin(t)} + \csc\left(\frac{\pi}{3}\right)$. Find $f'(t)$.

(c) Let $q(x) = \ln(x)\cos(x^2 - cx)$, where c is a constant. Find $q'(x)$.

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5. (30 points.) A large spherical snowball was melting. (A child took it inside the house at 11:00 am, and his parents had not yet noticed.) At 11:07 am, its radius was 30 cm, and was decreasing at $\frac{1}{3}$ cm per minute. Was its surface area increasing or decreasing? At what rate? (Be sure to include the correct units in your answer.)

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6. A fire-breathing monster is thrown upwards on the planet Yuggxth. Its height t seconds after it is thrown is $40t - 5t^2$ feet, until it hits the ground again.

(a) (4 points) Is the monster falling or rising 6 seconds after being thrown? How fast?

(b) (7 points) How long after being thrown does the monster reach its maximum height?

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7. (15 points) If $y^3 = \sin(7x-y) - \sqrt{2}$, find $\dfrac{dy}{dx}$ by implicit differentiation. (You must solve for $\dfrac{dy}{dx}$.)

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8. (17 points) Suppose we know the following about the function h:

(1) h is defined and continuous on $(-\infty, \infty)$, and $h'(x)$ and $h''(x)$ exist on all of $(-\infty, \infty)$.
(2) h has only one critical number, namely 1.
(3) $h(1) \approx -2.718$.
(4) $h'(x) < 0$ for x in the interval $(-\infty, 1)$.
(5) $h'(x) > 0$ for x in the interval $(1, \infty)$.
(6) The only solution to $h''(x) = 0$ is $x = 0$.
(7) $h(0) = -2$.
(8) $h''(x) < 0$ for x in the interval $(-\infty, 0)$.
(9) $h''(x) > 0$ for x in the interval $(0, \infty)$.
(10) $\lim_{x \to -\infty} h(x) = 0$.

Find the asymptotes, intervals of increase and decrease, local minimums and maximums, intervals of concavity up and down, and inflection points. Then draw the graph of h. Make sure that the graph matches the information about concavity etc. that you found.

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9. (35 points.) Consider a flat square sheet of cardboard that is 6 feet wide and 6 feet tall. You cut out a squares of equal size out of each corner of the sheet. Then you fold the remaining sides (flaps) of the cardboard up to make a rectangular box (with no top). If you want to create box with the largest volume, what is the side length of each square that is cut out?

Be sure to verify that your maximum or minimum really is what you claim it is.

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3. Extra Sample Problems for Final Exam

These extra problems are mostly on material since the second midterm, or are slightly different problem types for earlier material. They are not representative of how much of the final exam will be on each part of the course. See the sample Midterm 1 and sample Midterm 2.

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10. (6 points.) Let $w(t)$ be the water flow at time t in a river at a particular measuring station. Assume that t is measured in days, and that $w(t)$ is measured in m^3/sec. During the beginning of the rainy season, do you expect $w'(t)$ to be positive or negative? Why?

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11. (14 points.) You are told that $f(x)$ is a function defined for $x \neq -1$ whose derivative is $f'(x) = \dfrac{x^2 + 2x - 3}{(x+1)^2}$. Find the critical points of $f(x)$ and identify each of them as a local minimum, local maximum, or neither.

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12. (7 points) The derivative of the function $f(x) = (x-3)e^{-x}$ is given by $f'(x) = -(x-4)e^{-x}$, and the second derivative is given by $f''(x) = (x-5)e^{-x}$. Determine whether f has a local minimum, a local maximum, or neither at $x = 4$.

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13. (18 points.) You want to construct a cylindrical water tank can, with no top. Your budget allows for the purchase of 900 square feet of material. What is the maximum possible volume?

Set up, but **do not attempt to solve**, the appropriate maximization or minimization problem. That is, give a function $f(x)$ (not necessarily calling the function f or the variable x) for x a suitable quantity related to the problem (say what it actually is!), give a suitable domain, and say whether you want to maximize or minimize your function on this domain. Provide justification for all steps (possibly including a picture).

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14. (9 points) Let g be a function such that $g(2) = 3$ and whose derivative is known to be $g'(x) = -\sqrt{2x}e^{x-2}$. (You are not given a formula for g. Don't try to guess one–you won't succeed.) Use the linearization (tangent line approximation) to estimate the value of $g(1.9)$.

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15. (8 points) The function $f(x)$ satisfies the following three properties:

$$f(7) = 4, \qquad f'(7) = 2, \qquad \text{and} \qquad f''(7) = -3.$$

Use the linearization (tangent line approximation) to estimate the value of $f(6.97)$. (You won't need to use all the information given.)

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16. (10 points.) Suppose f is continuous on $[-1, 7]$, and $f'(x)$ exists and satisfies $-2 < f'(x) < 3$ for all x in $(-1, 7)$. Give the best possible estimate on $f(7) - f(-1)$. Justify your answer.

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17. (10 points) Find the exact value of $\lim_{x\to 0} \dfrac{\arctan(ax)}{\tan(x)}$ (possibly including ∞ or $-\infty$), where a is a constant. Or explain why the limit does not exist or there is not enough information to evaluate it. Give justification (not just heuristic arguments).

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18. (12 points) Consider the curve given by $x^2 - 3xy + y^2 = -1$. Find the equation of the tangent line to this curve at the point $(2, 1)$.

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19. (8 points) Set $h(x) = \sin(\pi + x^2)$. At $x = 0$, does h have a local minimum, local maximum, or neither?

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20. (8 points) Set $q(x) = \dfrac{x}{1 + x^2}$. At $x = 0$, does q have a local minimum, local maximum, or neither?
