Math 251 Final exam worksheet NAME: PROBLEM LIST
Th 13 March 2025 Student ID:________________________

Not all types of problems that will appear on the final exam are here. In particular, see the separate sample final exam for other possibilities, but that is not the only place you need to look. For example, a problem on the real final exam like Problem 1 here might not give you the derivatives of the function, and a problem like Problem 3 here might only ask you to set up the problem.

The point values are rough.

1. (15 points) Let $g(x) = 128x - 8x^3 + x^4$. Its first two derivatives are

$$g'(x) = 128 - 24x^2 + 4x^3 = 4(x-4)^2(x+2) \qquad \text{and} \qquad g''(x) = -48x + 12x^2 = 12x(x-4).$$

(You do **not** need to check these.)

Find the open intervals of increase and decrease, values of x at which critical points occur, values of x at which local minimums occur, values of x at which local maximums occur, open intervals of concavity up and down, and values of x at which inflection points occur.

2. (11 points/part.) Find the exact values of the following limits (possibly including ∞ or $-\infty$), or explain why they do not exist (not even being ∞ or $-\infty$) or there is not enough information to evaluate them. Give justification in all cases (not just heuristic arguments).

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 - e^x - 6}{x-2}$.

(b) $\displaystyle\lim_{\varphi\to 0} \frac{\cos(2\varphi)}{\varphi}$. (Be sure to show your work!)

(c) $\displaystyle\lim_{x\to 0} \frac{e^{k\sin(x)} - 1}{\sin(x)}$, where k is a nonzero constant.

3. (25 points.) An open rectangular box (no top) is to have a base that is twice as long as it is wide. 96 square feet of material are available to make the box. Find the dimensions of the box which maximizes the volume.

Include units, and be sure to verify that your maximum or minimum really is what you claim it is.

4. (15 points) Find $\dfrac{dy}{dx}$ if $xy = \tan(y-x) + \ln(2)$. (Use implicit differentiation. You must solve for $\dfrac{dy}{dx}$.)

5. (10 points/part) Differentiate the following functions.

(a) $w(t) = \dfrac{\cos(t)}{3t} - \dfrac{5}{\sqrt{t}} + e^2$.

(b) $h(t) = \pi^2 - 3t^2\cos(t)$.

(c) $w(t) = \arctan\left(\sqrt{t}\right) - \ln(2\pi)$.

6. (22 points) A certain section of the San Andreas Fault runs straight north-south. On 1 January 1997, the west side was moving north (relative to the east side) at 3 cm/year (0.03 meters/year). At the same time, the town of Hicksville was 2 km (2000 meters) west of the fault, and the town of Gorman was 1 km (1000 meters) east of the fault and 4 km (4000 meters) farther south than Hicksville. Were these two towns getting closer together or farther apart at this time? At what rate?

7. (10 points) You have just removed a pizza from a hot oven. Let $T(t)$ be the temperature of the pizza in degrees Fahrenheit at time t minutes after removing it from the oven. If $T(5) = 400$ and $T'(5) = -6$, use the linear approximation (tangent line to $y = T(t)$ at $t = 5$) to estimate $T(10)$.

8. (15 points; point values of parts as shown) A small spacecraft takes off from the surface of a planet, reaches a maximum height, and then crashes. Its position at time t is given by $y(t) = 9t^2 - 4t^3$, where $y(t)$ is measured in kilometers (km) above the surface and t is measured in minutes (min). Answer the following questions, being careful to give correct units when called for.

(a) (5 points) Find the upwards velocity of the spacecraft at time $t = 1$.

(b) (5 points) Find the average upwards velocity of the spacecraft between time $t = 0$ and time $t = 2$.

(c) (5 points) How long will it take for the spacecraft to reach its maximum height?

9. (15 points.) Let $g(x) = x^3 - 15x^2 + 16$. Use the methods of calculus to find the exact values of x at which has its maximum and minimum values on the interval $[-1, 1]$.

(No credit will be given for correct guesses without supporting work that is valid for general functions of the sort considered in this course.)