Harmonic and Functional Analysis of Frames
Math 685, Winter 2022

Class Time:	MWF 9-9:50a.m. in 117 Fenton Hall
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Background and goals. A frame is a generalization of the concept of a basis to sets which are overcomplete. That is, frame expansions are in general not unique and instead they satisfy a certain stability condition. Although frames were introduced in 1950's, this area has experienced a renewed interest in recent years with the advent of wavelets. In this course we plan to explore the following topics depending on the interest of students.

- General frames and Riesz bases in Hilbert spaces: dual frames, canonical dual frames, Naimark's dilation theorem.
- Frames in finite dimensional spaces: equiangular frames, fusion frames, connections with algebraic combinatorics and Littlewood-Richardson tableaux.
- Frames in infinite dimensional spaces: Kadison's Pythagorean Theorem, characterization of frame norms with prescribed frame operator and the Schur-Horn theorem.
- Frames and Riesz bases in shift-invariant spaces.
- The solution of the long standing Kadison-Singer problem and its equivalent formulation in terms of the paving conjecture, the Feichtinger conjecture, and the Bourgain-Tzafriri conjecture. The ramifications of this solution to the frame theory.

Prerequisites. Math 616/7/8 Real Analysis.

Grading. There will be a couple of homework assignments. There will be no exams.

Textbook. The standard reference is a textbook by O. Christensen, *An introduction to frames and Riesz bases*, Birkhäuser 2002. However, many results covered in this course are taken from more recent research papers.

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