MATH 684/5/6, FOURIER ANALYSIS AND WAVELETS
FALL 2007, WINTER 2008, AND SPRING 2008

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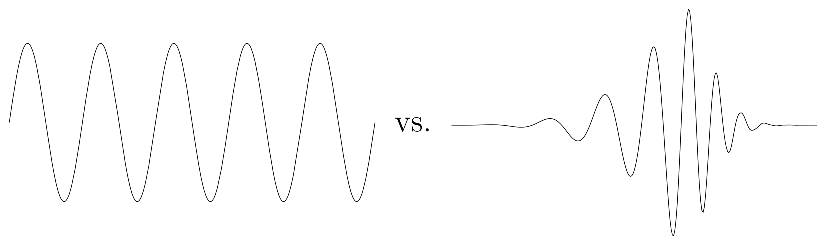

1. Syllabus

1.1. **Background and goals.** Fourier analysis is a subject of mathematics that originated with the study of Fourier series and integrals. Nowadays, Fourier analysis is a vast area of research with connections and applications in various branches of science including partial differential equations, potential theory, mathematical physics, number theory, signal analysis, and tomography. A recent noteworthy area of focus in Fourier analysis are wavelet bases. The theory of wavelets is a very active area of research with many real-world applications to signal processing, e.g. JPEG 2000 image compression algorithm.

This course is an introduction to Fourier analysis and wavelets. There will be 3 main themes corresponding to 3 terms:

(i) classical Fourier series and transforms,
(ii) construction and L^2 theory of wavelets,
(iii) wavelets in function spaces.

While all of these themes are closely connected, I will try to keep a clear distinction between these areas. That way, each term we can have a fresh start without assuming much of what was done before. And as the term progresses the topics will get more technical. More specifically, we are planning to cover the following topics.

1.2. **Fall 2007.**

- Fourier series, Fejér summability, and pointwise convergence,
- Rates of decay of Fourier coefficients,
- Fourier series in L^2, the Parseval Theorem,
- Fourier-Stieltjes series, the Herglotz Theorem,
- Convergence in norm, pointwise convergence and divergence,
- Interpolation theorems of Hausdorff-Young and Marcinkiewicz,
- Fourier transform in $\mathbb{R}^n$, Fourier inversion theorem, and Plancherel's theorem,
- Multiple Fourier series and the Poisson summation formula,
- Schwartz class and tempered distributions.

1.3. Winter 2008.

- General expansions in Hilbert spaces, Riesz bases, and frames,
- Shift-invariant spaces and the spectral function,
- Multiresolution analysis (MRA), scaling functions, and wavelets,
- Minimally supported frequency (MSF) wavelets and wavelet sets,
- Construction of (spline) Strömberg wavelets, and Meyer wavelets,
- Compactly supported Daubechies wavelets,
- Wavelets in higher dimensions, expansive dilations,
- Characterizations in the L^2 theory of wavelets, basic equations,
- Generalized multiresolution analysis (GMRA) and the wavelet dimension function.

1.4. Spring 2008.

- Hardy-Littewood maximal function and the Hilbert transform,
- Calderón-Zygmund decomposition and singular integral operators,
- Hardy spaces on $\mathbb{R}^n$ and atomic decompositions,
- Littlewood-Paley decomposition, Besov and Triebel Lizorkin spaces,
- Wavelet bases and frames in function spaces, sequence spaces of wavelet coefficients,
- Applications to signal processing, discrete Fourier and wavelet transforms.

1.5. **Prerequisites.** Math 616/7/8 Real Analysis.

1.6. **Grading.** There will be a couple of homework assignments each term. There will be no exams. In Spring term each student will give an oral presentation on a topic of his/her choice related to this course.

1.7. **Textbooks.** I will not follow any book for a longer period of time. Instead, I will use parts of the following books: [4, 6] in Fall, [1, 3, 8] in Winter, and [2, 5, 7] in Spring term.

References

[1] I. Daubechies, *Ten lectures on wavelets*, SIAM, Philadelphia, PA, 1992.
[2] L. Grafakos, *Classical and modern Fourier analysis*, Pearson Education, 2004.
[3] E. Hernández, G. Weiss, *A first course on wavelets*, CRC Press, 1996.
[4] Y. Katznelson, *An introduction to harmonic analysis*, Cambridge Univ. Press, 2004.
[5] Y. Meyer, *Wavelets and operators*, Cambridge Univ. Press, 1992.
[6] M. Pinsky, *Introduction to Fourier Analysis and Wavelets*, Brooks/Cole 2002.
[7] E. Stein, *Harmonic analysis: Real-variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton Univ. Press, 1993.
[8] P. Wojtaszczyk, *A Mathematical Introduction to Wavelets*, Cambridge Univ. Press, 1997.