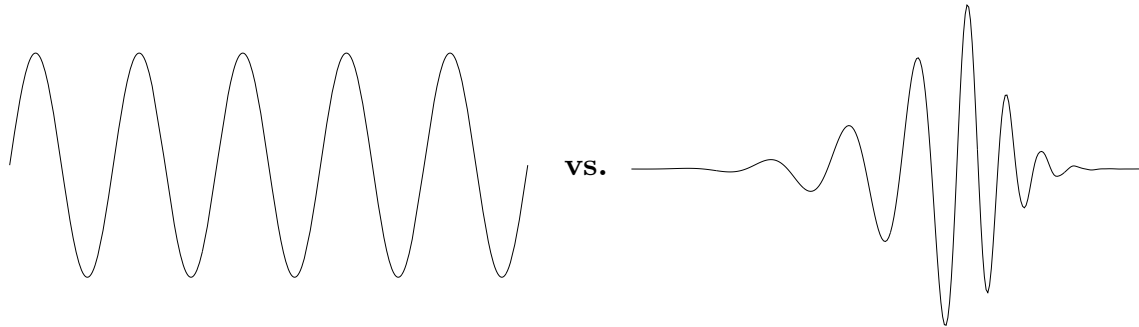# Math 684 in Fall 2004 and Math 685 in Winter 2005
Advanced Topics in Analysis—Introduction to Fourier Analysis and Wavelets

Marcin Bownik

Fourier analysis is a subject of mathematics that originated with the study of Fourier series and integrals. Nowadays, Fourier analysis is a vast area of research with applications in various branches of science including signal analysis, tomography, partial differential equations, potential theory, mathematical physics and number theory.

A recent noteworthy area of focus in Fourier analysis is orthogonal expansions in wavelet bases. The theory of wavelets is a very active area of research with many real-world applications.

This course is an introduction to the theory of Fourier series, Fourier integrals, wavelets, and related topics. More specifically, we are planning to cover the following:

1) General properties of orthogonal systems, Riesz bases, and frames.
2) Convergence and summability of Fourier series. Intro to lacunary series.
3) Fourier transforms of L^2 functions, inversion formula, Plancherel's theorem.
4) Multivariable Fourier series, the Poisson summation formula.
5) Theory of distributions, Fourier transforms of tempered distributions, the Paley-Wiener theorem.
6) General theory of wavelets, scaling functions, multiresolution analysis.
7) Construction of Strömberg wavelets, Meyer wavelets, and compactly supported Daubechies wavelets.
8) Multivariable wavelets, tensor products.
9) Wavelets and Calderón-Zygmund operators in various function spaces.
10) Applications to signal processing, discrete Fourier and wavelet transforms.

Prerequisites: Real Analysis Math 616, 617, 618.

Grading: There will be a couple of homework assignments. Since there will be no final exam each student will give an oral presentation on a subject of his/her choice related to this course.

Textbooks: Y. Katznelson, *An Introduction to Harmonic Analysis*, Dover, 1976. P. Wojtaszczyk, *A Mathematical Introduction to Wavelets*, Cambridge Press, 1997.