

Brief Article

The Author

July 20, 2026

1 Lattices and the Weyl Group

The heart of this book is Weyl's character formula giving the characters of all irreducible representations for any compact group.

Let us focus on the key case when $\mathcal{G}$ is simple with negative-definite Killing form and G has a corresponding left and right invariant metric. If T is the maximal torus of G , each element of G is conjugate to an element of T , and consequently the characters of the group are completely determined by their values on a maximal torus T . The universal cover of this torus is R^n , so the characters are straightforward functions on R^n that are invariant under the lattice $\mathcal{L}$ with $T = R^n/\mathcal{L}$. If we ignore the invariant metric, this lattice is just Z^n , but this metric plays a key role in the theory and cannot be ignored. When we take it into account, we get more interesting pictures of the lattices, like those shown below.

