Azimuthal Anisotropy without Hydrodynamics

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Azimuthal anisotropy in heavy-ion collisions is studied without the use of hydrodynamics whose success has relied on the assumption of fast thermalization that may not be valid. Ridges generated by semi-hard scattering of intermediate-momentum partons can be sensitive to the initial spatial configuration of the medium in non-central collisions. In a simple treatment of the problem where only thermal partons are considered, analytical formulas can be derived that yield results in accord with the data on v_2 for p_T <1-2 GeV/c. For higher p_T, shower partons from high-p_T jets must be included, but they are not considered here. The success of this approach provides an alternative to the usual notion of elliptic flow and avoids the questionable assumption of rapid equilibration for hydrodynamical flow to be applicable at early times.

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The use of relativistic hydrodynamics in the study of particle production in heavy-ion collisions at RHIC has led to claims of success for $p_T \lesssim 1.5$ GeV/c, provided that the evolution of a thermalized system starts at $\tau_0 = 0.6$ fm/c [1]. Such a short thermalization time is hard to understand in QCD; whatever the mechanism, it has led to the view that the system under study is a strongly interacting quark-gluon plasma. A strong phenomenological support for such a view is that the observed azimuthal anisotropy in the momenta of the emitted particles can best be interpreted by the rapid development of a large pressure gradient on the short side of the overlap region before the spatial anisotropy disappears due to radial expansion [2]. The absence of viscosity in the hydrodynamical equations further led to the description of the system as a perfect liquid [3]. However, the whole structure of that line of interpretation depends on the validity of the short thermalization time. One could take an alternative line on the basis that the short thermalization time is unrealistic by any understandable physical mechanism and that the application of hydrodynamics is unreliable for a significant part of the evolution process. The burden of such a view is to provide another way to understand the azimuthal anisotropy (or v_2) without hydrodynamics. The aim of this paper is to give a preliminary description of such a way.

Stimulated by the possibility that non-flow can be important in the STAR data [4, 5], we have studied a simple mechanism that can give rise to the observed azimuthal variation of the produced particles without hydrodynamics. We study the single-particle $dN/p_T dp_T d\phi$, when high-p_T jets are not important; from that distribution we derive an analytical form for $v_2(p_T)$ for any centrality. The source of the ϕ dependence is the enhanced thermal partons along the boundary of the system generated by semi-hard partons at the early stage of the collision. The hadrons formed from similar partons in the case of jet production selected by specific trigger condition have been referred to collectively as the ridge. We extend the notion of ridge to the situation where the momenta are low and the ridges are numerous without triggers.

We restrict our attention in this paper to the low p_T region with $p_T < 2$ GeV/c. The hadronization process will be treated by recombination, as we have done in the region $p_T > 2$ GeV/c [6]. In the higher p_T region the shower partons from high-p_T jets must be considered; their contribution to v_2 will be investigated in a future study. That region is not germane to the issue at hand of finding an alternative to the hydrodynamical approach that claims validity up to $p_T \sim 1.5$ GeV/c only. At low p_T only thermal partons are relevant. The ridges we consider are generated by low-momentum jets that are produced much more copiously than the high-momentum jets. The formation of ridges with or without identifiable peaks has been studied before in the recombination model [7]. We now apply similar consideration to the study of azimuthal anisotropy.

Let us adopt the very simple description of nuclear geometry with the assumption that the colliding nuclei have sharp boundaries at the spherical surface of radius R_A. At impact parameter b the almond-shaped overlap region in the transverse plane is bounded by two circular arcs whose angular ranges are

$$|\phi| < \Phi \quad \text{and} \quad |\pi - \phi| < \Phi, \tag{1}$$

where ϕ is the azimuthal angle measured from the centers of the two nuclei, respectively, and

$$\Phi = \cos^{-1}(b/2R_A). \tag{2}$$

That is the spatial configuration of the system under study at the initial time immediately after collision, even though the particles to be detected are formed significantly later with momenta measured by the same azimuthal angle ϕ. We do not assume that thermalization of the partons takes place rapidly, so hydrodynamical description of the ensuing expansion (which undoubtedly takes place) may not be applied at early times. But we do assume that the partons are thermalized by the time of hadronization at least 10 fm/c later, so that they can be described by some thermal distribution.

The issue concerning the ϕ dependence of the detected hadrons is about the physics that is sensitive to the initial configuration of the system. If one focuses only on very soft particles, it is hard to relate the ϕ anisotropy of the energy-momentum tensor to the initial configuration without starting out with ϕ-dependent pressure gradient that requires fast equilibration [1, 2]. We therefore consider semi-hard scattering that is soft enough to have frequent occurrences but hard enough to create intermediate-p_T jets at early times. If those jets are in the 2-3 GeV/c range, they can strongly influence the parton distribution at $p_T < 2$ GeV/c which is the region of our concern here. The corresponding time scale is $\tau < 0.1$ fm/c, short enough to be sensitive to the initial spatial configuration. There is no reliable way to calculate those weak jets, but that does not mean that they are not important. To have precise calculational schemes at the hard (pQCD) and soft (hydrodynamics) ends does not guarantee the importance of the physical consequences.

In events triggered by high-p_T particles it has been found that jet structure consists of a prominent peak above a ridge in the $\Delta\eta$ distribution [8]. Quantitative study of the relative strength of the peak (denoted as Jet, or J) and the ridge (R) shows that J/R becomes small when the trigger momentum is low and the associated-particle momentum is even lower ($<$ 2 GeV/c) [9, 10]. When the ridge yield dominates over the Jet yield, the underlying jet is hardly recognizable and has been referred to as the phantom jet [11]. The ridge has been described as the recombination product of enhanced thermal partons due to the energy lost by the semi-hard parton traversing the medium [7]. The ridge being in the immediate vicinity of the jet direction implies that the semi-hard scattering takes place near the surface of the medium and that the thermalization of the energy lost is restricted to the region near the jet trajectory.

Applying the notion of phantom jets to our present problem, we have a collection of those semi-hard jets on the surface of the almond-shaped region, if the virtuality of the jets is small enough so that many of them are produced in a nuclear collisions. At $\eta \sim 0$ the semi-hard scattering involves low-x (< 0.03) partons, which are abundant. The direction of a scattered parton is random, but in any given patch of spatial region near the surface the average direction of all outward partons is normal to the surface, since that is the only direction in the geometry of the problem. When integrated over the surface, there is a layer of emitters of semi-hard partons whose directions are prescribed by the geometry of the overlap at the initial time. The ridge of enhanced thermal partons may develop subsequently, after allowing some time for local thermalization to take place near the surface. But the average directions of the phantom jets are specified by Eq. (1), independent of the time it takes for hadrons to form. Since those phantom jets are the products of semi-hard partons directed outwards, the recoil partons directed inward are absorbed by the medium and contribute to the thermalization of the bulk that has no preferred direction of expansion in the transverse plane. In the present study we ignore any aspect of the longitudinal expansion and consider only hadrons at mid-rapidity. In summary the system under study can be described by two main types of thermal partons that are generated by the collision: (a) the bulk that is isotropic in ϕ, and (b) the ridges that is nonzero in the ϕ region specified by Eq. (1).

In Ref. [6] it is shown how the pion distribution can be obtained from the parton (quark and antiquark) distribution in the recombination model. The region of interest there is for $p_T > 2$ GeV/c, so shower partons are important in addition to the thermal partons. We now use the same formalism for $p_T < 2$ GeV/c and consider only the thermal partons. For the bulk medium the invariant distribution of the thermal partons is

$$q_0 \frac{dN_q^B}{dq_T d\phi} = C q_T e^{-q_T/T} \tag{3}$$

where, with our attention focused on only the momentum in the transverse plane at $y = 0$, q_T is the parton transverse momentum and q_0 is its energy. The value of C is of no concern here since it will be canceled in a ratio to be calculated. Adopting the same formalism of recombination described in [6], we have for the pion distribution with the pion mass being neglected

$$B(p_T) = \frac{dN_\pi^B}{p_T dp_T d\phi} = \frac{C^2}{6} e^{-p_T/T}, \tag{4}$$

which describes the observed low-p_T behavior of the bulk (B) medium having the parton distribution given by Eq. (3). In (4) there is no ϕ dependence.

According to our discussion above, for ϕ satisfying Eq. (1) there is in addition the contribution from the enhanced thermal partons arising from the energy loss of the semi-hard partons that initiate numerous phantom jets. There is no reliable scheme to calculate the semi-hard scattering process of low-x partons, the radiative process that they undergo while traversing the medium, and the enhanced thermalization of the partons in the vicinity of each trajectory. We summarize the effect of all those processes by one parameter T' and write

$$q_0 \frac{dN_{q_T}^{B+R}}{dq_T d\phi} = C q_T e^{-q_T/T'} \Theta(\phi), \tag{5}$$

where

$$\Theta(\phi) = \theta(\Phi - |\phi|) + \theta(\Phi - |\pi - \phi|), \tag{6}$$

$\theta(x)$ being the Heavyside step function. The resulting pion distribution for both the bulk (B) and the ridge (R) is then

$$B(p_T) + R(p_T, \phi) = \frac{dN_\pi^{B+R}}{p_T dp_T d\phi} = \frac{C^2}{6} e^{-p_T/T'} \Theta(\phi). \tag{7}$$

Without any triggers it is not feasible to isolate the ridge contribution experimentally, as it has been done in the study of two-particle correlation, and to determine T' directly [8–10].

From Eqs. (4) and (7) we obtain

$$R(p_T,\phi) = R(p_T)\Theta(\phi),$$
$$R(p_T) = \frac{C^2}{6} e^{-p_T/T'}\left(1 - e^{-p_T/T''}\right), \qquad (8)$$

where

$$\frac{1}{T''} = \frac{1}{T} - \frac{1}{T'} = \frac{\Delta T}{TT'}, \qquad \Delta T = T' - T. \qquad (9)$$

Experimental study of the ridges in triggered events in central Au+Au collisions shows that ΔT is in the range of 40-50 MeV [9]. We take $\Delta T = 45$ MeV in the following.

The second harmonic in the ϕ distribution is

$$v_2(p_T) = \langle \cos 2\phi \rangle = \frac{\int_0^{2\pi} d\phi \cos 2\phi \; dN/p_T dp_T d\phi}{\int_0^{2\pi} d\phi \; dN/p_T dp_T d\phi}. \quad (10)$$

When we substitute for $dN/p_T dp_T d\phi$ the general expression in terms of $B(p_T)$ and $R(p_T,\phi)$, we obtain

$$v_2(p_T) = \frac{\int d\phi \cos 2\phi R(p_T,\phi)}{\int d\phi \left[B(p_T) + R(p_T,\phi)\right]} = \frac{R(p_T)\sin 2\Phi}{\pi B(p_T) + 2\Phi R(p_T)}. \qquad (11)$$

To exhibit the separate dependence on p_T and b, let us write

$$v_2(p_T,b) = \frac{\sin 2\Phi(b)}{\pi B(p_T)/R(p_T) + 2\Phi(b)} \qquad (12)$$

so that when Eq. (8) is rewritten in the case of pion as

$$R(p_T) = \frac{C^2}{6} e^{-p_T/T}\left(e^{p_T/T''} - 1\right), \qquad (13)$$

we have

$$B(p_T)/R(p_T) = \left(e^{p_T/T''} - 1\right)^{-1}. \qquad (14)$$

For small p_T the above ratio is large, so the second term in the denominator of Eq. (12) is small by comparison, and we obtain

$$v_2^\pi(p_T,b) \simeq \frac{p_T}{\pi T''}\sin 2\Phi(b). \qquad (15)$$

This is a simple formula that describes both the p_T and centrality dependences at small p_T, the latter being through Eq. (2).

Note that $R(p_T)$ in Eq. (13) is proportional to $B(p_T)$ in Eq. (4), so C^2 as well as the exponential factor are canceled out in Eq. (14). Such factors can depend on centrality, but are irrelevant to $v_2^\pi(p)$. T'' may have some centrality dependence, but it would be mild compared to that of the factor $\sin 2\Phi(b)$ that depends on b strongly and explicitly. The peak of $\sin 2\Phi(b)$ at $\Phi = \pi/4$ corresponds to $b = \sqrt{2}R_A$, which in turn corresponds to ∼50% centrality for any colliding nuclei. The data indeed shows a maximum of v_2^π at that centrality for small p_T [12] (see Fig. 1). From the π^+ distribution at 40-50% [13], a fit of the exponential behavior in the range $0.5 < p_T < 2.7$ GeV/c gives $T = 0.287$ GeV. Using that value of T and 0.045 GeV for ΔT, we obtain from Eq. (9)

$$T'' = 2.12\,\text{GeV}. \qquad (16)$$

It then follows from Eq. (15) that at small p_T the maximum value of v_2^π (at 50% centrality), divided by p_T, is

$$\frac{v_2^\pi(p_T,b)}{p_T} = \frac{1}{\pi T''} = 0.15\ (\text{GeV/c})^{-1}, \qquad (17)$$

which agrees very well with the data in Fig. 1 that shows $v_2 = 0.075$ at $p_T = 0.5$ GeV/c. This is the first quantitative demonstration that v_2 and ridge phenomenologies are related. For $p_T > 0.5$ GeV/c, the small p_T approximation of $v_2^\pi(p_T,b)$ given in Eq. (15) is not good, so the full expression in Eq. (14) should be used in (12). The result is shown by the thick line in Fig. 1 for 50% centrality, and agrees well with the data [12] indicated by the blue stars for 40-50% centrality.

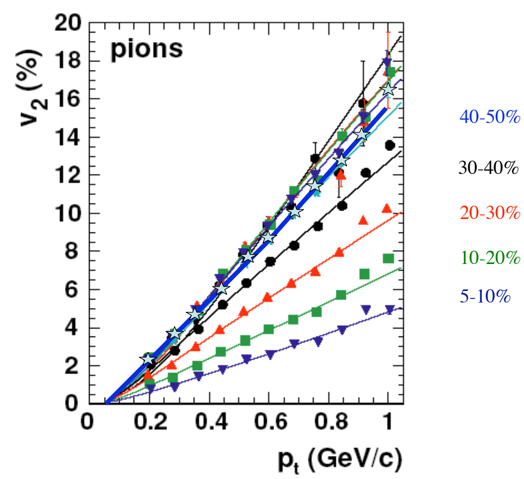

FIG. 1: (Color online) Calculated $v_2^\pi(p_T)$ for 50% centrality is indicated by the thick blue line. The data are from [12] for Au-Au collisions at 200 GeV, together with the light lines from that reference.

For proton production the same formalism of recombination applies, as for pion, and the ridge consideration is the same. To take the mass effect into account we replace all exponential forms, such as $e^{-p_T/T}$ for the bulk, by $\exp[-(m_T - m)/T]$, where $m_T = (p_T^2 + m^2)^{1/2}$ and m the proton mass. It then follows that at small p_T the v_2 formula for proton is

$$v_2^p(p_T,b) = \frac{p_T^2}{2\pi m T''}\sin 2\Phi(b). \qquad (18)$$

With the same T'' as in Eq. (15) we have universal behavior of the slope of $v_2^h(p_T,b)$ versus the transverse kinetic energy, E_K, which is p_T for pion and $p_T^2/2m$ for proton

$$\left.\frac{\partial v_2^h(p_T,b)}{\partial E_K}\right|_{p_T<0.5} = \frac{1}{\pi T''}\sin 2\Phi(b) \tag{19}$$

that is independent of the hadron type h. This universality is a consequence of T'' being a property of the partons in the ridges before hadronization. Thus it trivially satisfies the constituent quark scaling at low p_T, for which both v_2^h and E_K are scaled by the quark number n_q.

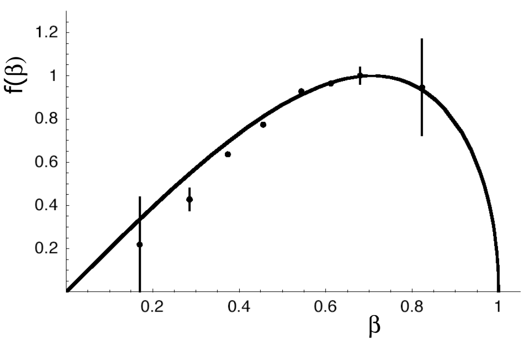

FIG. 2: Centrality dependence of v_2 normalized to 1 at maximum. Data are v_2 for Au-Au collisions at 130 GeV [14], scaled to unity at b=10fm.

If we define the scaled impact parameter to be $\beta = b/2R_A$, the explicit factor that describes the centrality dependence is

$$f(\beta) = \sin 2(\cos^{-1}\beta), \tag{20}$$

which is plotted in Fig. 2. It agrees with the Au-Au data on v_2 [14], normalized to unity at maximum. Since $f(\beta)$ does not depend on R_A explicitly, we expect it to agree with the Cu-Cu data also.

For $\beta > 0.7$, Eq. (18) is inadequate to describe proton production in more peripheral collisions because less thermal partons render the joint uud-distribution not factorizable and the effective value of T in the bulk is reduced. That leads to a decrease of $B(p_T)/R(p_T)$ and the second term in the denominator of Eq. (12) cannot be ignored. The consequence is that $v_2^p(p_T,\beta)$ continues to increase for $\beta > 0.7$. For the main range $0 < \beta < 1/\sqrt{2}$ our simple treatment described above is adequate to reproduce both the p_T and centrality dependences.

The next question concerns the dependence of T'' on energy and density. Properties of the ridge have so far been studied with the help of triggers, and our knowledge about ridges with unspecified semi-hard parton momenta is rudimentary. A reasonable first approximation of T' is to expand T'/T in terms of T on the basis that the bulk T encapsules the various separate dependences on energy and $N_{\rm part}$. If we write $T'/T = 1 + aT$, ignoring higher order terms, then we find from Eq. (9) that $a = 1/T''$ to the same order of approximation. Thus we expect that any sensitivity of T'' to energy and $N_{\rm part}$ would be of higher order. For that reason Eq. (19) is nearly a universal description of v_2 at low p_T.

The above consideration applies only at midrapidity. In forward region the semi-hard scattering involves partons at larger x, but not too large because of recombination [15]. Lower parton density means that there are less phantom jets, so the ridge effect is reduced with increasing η, resulting in diminishing v_2.

We have shown that the observed azimuthal anisotropy at low p_T can be reproduced by a consideration of the emission of hadrons from the ridges whose formation is due to semi-hard scattering of partons. No fast thermalization is required, and no hydrodynamical flow from early times is assumed. We do not exclude the relevance of hydrodynamics at some later stage of the expansion, but it is not needed to derive the main features of v_2. No assumption about viscosity is made. From the point of view of this approach there is no basis to infer that the QGP is strongly interacting or that it behaves as a perfect liquid.

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[1] P. F. Kolb, J. Sollfrank, and U. Heinz, Phys. Rev. C **62**, 054909 (2000).
[2] P. F. Kolb and U. Heinz, in *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004).
[3] M. Gyulassy, nucl-th/0403032; J. I. Kapusta, nucl-th/0705.1277.
[4] T. A. Trainor and D. T. Kettler, hep-ph/0704.1674.
[5] T. A. Trainor, arXiv/0708.xxxx.
[6] R. C. Hwa and C. B. Yang, Phys. Rev. C **70**, 024905 (2004).
[7] C. B. Chiu and R. C. Hwa, Phys. Rev. C **72**, 034903 (2005); ibid (to be published), nucl-th/0704.2616.
[8] J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **95**, 152301 (2005).
[9] J. Putschke (for STAR Collaboration), Nucl. Phys. A **783**, 507 (2007); nucl-ex/0701074.
[10] J. Bielcikova (for STAR Collaboration), Nucl. Phys. A **783**, 565 (2007); nucl-ex/0701047.
[11] R. C. Hwa, nucl-th/0701018.
[12] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **72**, 014904 (2005).
[13] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. C **69**, 034909 (2004).
[14] C. Adler *et al.* (STAR Collaboration), Phys. Rev. C **66**, 034904 (2002).
[15] R. C. Hwa and C. B. Yang, Phys. Rev. C **76**, 014901 (2007).