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Hadron Multiplicity in Saturation Physics with Recombination

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Abstract

The study of hadron multiplicity as predicted by saturation physics is extended to include parton recombination as the hadronization process. There is some uncertainty in relating the parton density derived in CGC to the density of quarks that recombine. From a phenomenological analysis done at RHIC energies with a particular choice of that relationship, it is possible to show that there is only a small difference between the use of local parton hadron duality and recombination, even when extrapolated to LHC energy. However, there seems to be more questions raised than answered in this first step taken to treat hadronization in a quantitative way.

1 Introduction

The phenomenological verification of the basic ideas of parton saturation [1, 2] has been successfully demonstrated, not only in the dependences of hadron multiplicities on energy, centrality and rapidity in nucleus-nucleus collisions [3, 4, 5], but also in deuteron-nucleus collisions [6] where the results of the wounded-nucleon model [7] can be derived. Subsequently, extrapolation to higher energy, as at LHC, has been made for pp, pA and AA collisions [8]. While the agreement with existing data is impressive, and the prediction for LHC will be checked in just a few years, there is one point that has not been treated with as much care as in all other parts of the program. That point is on the hadronization of partons. We shall present below some consideration based on parton recombination without modifying any part of the saturation physics itself.

Hadron multiplicities are dominated by particles produced at low p_T for all hadronic colliding systems, but especially in nuclear collisions. In Refs. [3, 4, 5, 8] the problem of hadronization is circumvented either by use of an empirical constant or by appeal to local parton hadron duality (LPHD) [9], which has some phenomenological support from the study of particle production in high-energy jets. There is no independent verification of that duality at low p_T in heavy-ion collisions. Thus the phenomenology of saturation physics is incomplete until the hadronization problem is properly treated so that its success is not based on a circular argument that avoids hadronization.

2 Hadron multiplicity in saturation physics

From the expression for the number of produced partons [10]

$$\frac{d^2N}{d^2bd\eta} = c\frac{N_c^2-1}{4\pi^2 N_c}\,\frac{1}{\alpha_s}Q_s^2 \tag{1}$$

and the relation of the saturation scale Q_s^2 to the gluon structure function of the nucleon $xG\left(x,Q_s^2\right)$ [1, 2]

$$Q_s^2 = \frac{8\pi N_c}{N_c^2-1}\alpha_s\left(Q_s^2\right)xG\left(x,Q_s^2\right)\frac{\rho_{\rm part}}{2} \tag{2}$$

where $\rho_{\rm part}$ is the density of participants in the transverse plane, Kharzeev and Nardi (KN) obtained [3]

$$\frac{dN}{d\eta} = cN_{\rm part}\,xG\left(x,Q_s^2\right). \tag{3}$$

Using $Q_s^2 = 2$ GeV^2 and $xG\left(x,Q_s^2\right) = 2$ at $x = 0.02$, and relying on the experimental data on hadronic rapidity density at $\sqrt{s} = 130$ GeV, it is found that

$$c = 1.23 \pm 0.20. \tag{4}$$

This number is sufficiently close to 1 to lead to the assertion in [3] that the parton to hadron transformation is consistent with LPHD. Although no explicit mention of the jet production is made in the above derivation, the physics of hadronization of minijet is implicitly assumed, both in the derivation of Eq. (1) and in subsequent phenomenology [6], as well as in the context in which LPHD is hypothesized [9]. This line of reasoning is sometimes reversed in stating that because there is phenomenological support for LPHD one can regard the parton multiplicity as approximately the hadron multiplicity, and ignore the hadronization problem, which was the original intent of LPHD.

So far the best experimental support for color glass condensate (CGC) [2] is in hadron multiplicity and rapidity density in dA and AA collisions at RHIC [3]-[6]. However, by relying on LPHD that is an assumption in jet physics (even if it is approximately valid at p_T high enough to justify perturbative calculations before hadronization), the phenomenology of CGC has a logical gap in relating what has been used in leptonic or hadronic processes to the gross features of heavy-ion collisions that depend heavily on the hadronization process.

3 Hadronization by recombination

Our concern is precisely on the point that the relevance of jet physics is questionable in processes where the dominant contributions to the parton and hadron multiplicities are at low p_T, less than 2 GeV/c. It has been shown in detailed study of hadronization in heavy-ion collisions that recombination of partons is far more important than fragmentation for $p_T < 5$ GeV/c [11, 12, 13]. Although for $p_T > 2$ GeV/c the shower partons from hard scattering can play a significant role [14], only thermal partons are important for $p_T < 2$ GeV/c. Furthermore, the p/π ratio is around 1 at $p_T \sim 3$ GeV/c, a phenomenon that cannot be explained by fragmentation, but is well reproduced by recombination [11, 12, 13, 14]. Thus the application of recombination to the hadronization problem in saturation physics is not only desirable, but necessary.

The relationship between the parton and hadron distributions is rather simple in the formalism of the recombination model (RM) in [14]. The general formula for the distribution of a pion to be produced in the transverse plane with momentum p_T in the one-dimensional formulation per $d\eta d\phi$ is

$$p^0 \frac{dN_\pi}{dp_T} = \int \frac{dq_1}{q_1} \frac{dq_2}{q_2} F_{q\bar{q}}(q_1, q_2) R_\pi(q_1, q_2, p_T), \tag{5}$$

where the quark and antiquark momenta, q_1 and q_2, are collinear with p_T. $R_\pi(q_1, q_2, p_T)$ is the recombination function [14, 15]

$$R_\pi(q_1, q_2, p) = \frac{q_1 q_2}{p^2}\delta\left(\frac{q_1}{p} + \frac{q_2}{p} - 1\right). \quad (6)$$

For thermal partons the q and $\bar{q}$ distributions are statistical, so we may assume factorizability of $F_{q\bar{q}}(q_1, q_2)$

$$F_{q\bar{q}}(q_1, q_2) = \mathcal{T}_q(q_1)\mathcal{T}_{\bar{q}}(q_2). \quad (7)$$

If we let the invariant distribution of the partons have the form

$$\mathcal{T}_q(q_i) = q_i \frac{dN_q}{dq_i} = C q_i e^{-q_i/T}, \quad (8)$$

then we obtain by simple integration

$$\frac{dN_\pi}{p_T dp_T} = \frac{C^2}{6} e^{-p_T/T}, \quad (9)$$

which is in the exponential form for thermal pions, as observed at low p_T. The total numbers of quarks and pions are, respectively,

$$N_q = \int \frac{dq_i}{q_i}\mathcal{T}(q_i) = CT, \quad (10)$$

$$N_\pi = \int dp_T p_T \frac{dN_\pi}{p_T dp_T} = \frac{(CT)^2}{6}. \quad (11)$$

The quadratic dependence on CT in Eq. (11) is the distinctive characteristic of recombination of thermal partons in the formation of mesons. It would be $(CT)^3$ for the formation of proton. The ratio of the multiplicities of pions to quarks is then

$$r = \frac{N_\pi}{N_q} = \frac{CT}{6}, \quad (12)$$

which is proportional to the number of quarks.

Let us relate the above to the data on charged particle production. In Refs. [18, 19] the p_T spectra of identified particles are given for various centralities in Au+Au collisions at $\sqrt{s} = W = 130$ and 200 GeV. We focus on the π^+ data only and consider

$$J_{\pi^+}(p_T) \equiv (2/N_{\rm part})\, dN_{\pi^+}/dy\, p_T dp_T|_{y=0} \ . \tag{13}$$

Experimental fits of the data [18, 19] for $0.25 < p_T < 1.0$ GeV/c and 0-5% centrality yield

$$J_{\pi^+}(p_T, W = 130) = 39.6 \ \exp(-5.18\, p_T)({\rm GeV/c})^{-2}, \tag{14}$$

$$J_{\pi^+}(p_T, W = 200) = 37.4 \ \exp(-4.73\, p_T)({\rm GeV/c})^{-2}. \tag{15}$$

We identify Eq. (9) with (13), absorbing the normalization factor in the prefactor C^2, and obtain

$$C_{130} = 15.42\,{\rm GeV}^{-1}, \qquad T_{130} = 0.193\,{\rm GeV}, \tag{16}$$

$$C_{200} = 14.97\,{\rm GeV}^{-1}, \qquad T_{200} = 0.211\,{\rm GeV}, \tag{17}$$

Using Eqs. (10) and (11) leads us to

$$N_q(130) = 2.98, \qquad N_{\pi^+}(130) = 1.48, \tag{18}$$

$$N_q(200) = 3.16, \qquad N_{\pi^+}(200) = 1.67, \tag{19}$$

where N_q refers to the number of light quarks of any species, i.e., $u, d, \bar{u},$ or $\bar{d}$, not their sum. All gluons have been converted to $q\bar{q}$ pairs. These are the numbers of q and π^+ per unit y per participant pair, assuming the validity of the exponential forms in Eqs. (14) and (15) for all p_T. There has been some concern about the reduction of the number of degrees of freedom in a recombination process in which colored quarks become a colorless hadron. Such concerns are usually allayed by pointing out that soft gluon radiation can always carry away color without diminishing the momenta of the coalescing partons [16]. How such soft gluons recombine among themselves when the density is high and then hadronize is usually regarded as outside the scope of recombination models that attempt to calculate the momentum distributions of hadrons with non-vanishing momenta. The behavior of both the parton and hadron distributions in the region $0 < p_T < 0.25$ GeV/c are not known either experimentally or theoretically. Our extrapolation of Eqs. (8) and (9) into that region is an approximation that can be the source of some inaccuracy.

Because of our explicit mechanism of hadronization, the partons and hadrons that we refer to above are very specific. No gluons remain after conversion to $q\bar{q}$ pairs, since direct hadronization of gluons is unknown. All light quarks have the same multiplicity, $N_q, q = u, d, \bar{u}, \bar{d}$, and only charged pions are separately counted, i.e., $N_\pi = N_{\pi^+} = N_{\pi^-}$. From Eqs. (18) and (19) we get

$$r(130) = 0.50, \qquad r(200) = 0.53, \tag{20}$$

a small, but significant, increase with energy.

We cannot make direct comparison of the numbers in Eq. (20) with LPHD or with the value of c in Eq. (4), since neither parton nor hadron is explicitly defined there. If we want to relate all charged hadrons to the number of quarks, N_π should first be doubled to yield $N_{\pi^\pm} \equiv N_{\pi^+} + N_{\pi^-}$ and then increased by some factor κ to account for $K^\pm, p$ and $\bar{p}$. Such an adjustment can change r to a value for $N_{\rm ch}/N_q$ that is in the vicinity of c in Eq. (4). Thus, roughly speaking, the phenomenology of hadronization by recombination is not inconsistent with the phenomenology of satuation physics by KN at RHIC energies [3]. However, N_q may not be the number of partons referred to in KN. If a parton refers only to a gluon, which in turn is converted to $u\bar{u}, d\bar{d}$ or $s\bar{s}$, then the number of partons differs from N_q, as we shall consider in the next section. Color factors are irrelevant in recombination, since their presence in $F_{q\bar{q}}$ and R_π in Eq. (5) cancel, as discussed in [14].

4 Hadron multiplicity at LHC

In the calculation of hadron multiplicities at higher energies, various centralities and rapidities in [8] (hereafter referred to as KLN), the basic rule is that c (or its later variants) is treated as a constant, essentially on the basis of LPHD, so that all the other dependences are prescribed by the physics of CGC. Good agreement with RHIC data has been obtained [4, 6]. In extrapolating to LHC energy, an explicit formula is given, which for $y = 0$ takes the simple form [4, 5, 8] with $W = \sqrt{s}$

$$\frac{2}{N_{\rm part}} \frac{dN_{\rm ch}}{dy}\bigg|_{y=0} = \frac{c}{\alpha_s} = c\,\frac{9}{4\pi} L(W), \tag{21}$$

where

$$\begin{aligned} L(W) &= \left(\frac{W}{W_0}\right)^{\tilde{\lambda}} \ln\left[Q_s^2(A, W, y=0)/\Lambda^2_{\rm QCD}\right] \\ &= \left(\frac{W}{130}\right)^{0.252} \left[3.93 + 0.252\,\ln\left(\frac{W}{130}\right)\right]. \end{aligned} \tag{22}$$

The numerical factor $9c/4\pi$ has been given the value 0.87, which corresponds to c being very nearly the value given in Eq. (4), and fits the data at $W = 130$ GeV. The y dependence is also explicit, but we focus on the $y = 0$ point only here.

There are two parts to the right-hand side of Eq. (21): $(9/4\pi)L(W)$ is the number of the partons produced, and c represents the number of charged hadrons produced per parton. That is to be contrasted from the features of recombination that can be characterized by writing the rapidity density of pions produced as

$$H_{\pi^+}(W) = \int dp_T p_T \; J_{\pi^+}(p_T, W) = r(W) N_q(W), \tag{23}$$

where N_q is the number of quarks produced and r represents the number of π^+ (or π^-) produced per quark (and corresponding antiquark), as given in Eqs. (10)-(12). The main difference between c and r is that the latter is proportional to N_q and is energy dependent.

In extrapolating to the energy of LHC, we rely on KLN for the W dependence of $L(W)$, since our role here is only to attach recombination to the physics of CGC. However, before going to higher energy, we must first establish a connection between the partons considered in saturation physics and the quarks that recombine. They are not the same, as can be seen in the RHIC regime. Without involving hadronization, we can determine the ratio of parton production at $W = 130$ and 200 GeV and find $L(200)/L(130) = 1.15$, whereas the ratio of recombining quarks is $N_q(200)/N_q(130) = 1.06$, which follows from Eqs. (18) and (19). This discrepancy must be taken into account if extrapolation to 5.5 TeV is to be meaningful. To that end, let us adopt the connection formula

$$N_q(W) = \beta \; L(W)^{\gamma} \; . \tag{24}$$

From the ratios at the two values of W, we find

$$\gamma = 0.42 \; , \tag{25}$$

and from Eqs. (18) and (22) evaluated at $W = 130$ GeV, we obtain

$$\beta = 1.68 \; . \tag{26}$$

The value of γ that differs from 1 highlights the difference between the saturating partons and the recombining quarks.

We now combine Eqs. (12) and (23) and get for the recombination model

$$H^{\rm RM}_{\pi^+}(W) = \frac{1}{6} N_q^2(W) = 0.47\ L(W)^{0.84}\ . \tag{27}$$

This is to be compared with Eq. (21), which is rewritten here as

$$H^{\rm KLN}_{\rm ch}(W) = 0.87\ L(W)\ . \tag{28}$$

To generalize Eq. (27) to include all charged particles would involve factors that are not precisely known, especially at higher values of W. But we can check the ratio at $W = 130$ and 200 GeV with the data [20], which give

$$R^{\rm data}_{\rm ch} = \frac{H^{\rm data}_{\rm ch}(200)}{H^{\rm data}_{\rm ch}(130)} = \frac{3.78}{3.37} = 1.12\ . \tag{29}$$

From Eqs. (27) and (28) one gets

$$R^{\rm RM}_{\pi^+} = 1.12, \qquad R^{\rm KLN}_{\rm ch} = 1.15\ . \tag{30}$$

The errors on all those numbers are at the 5% level, so they all agree within errors.

If we boost the normalization of $H^{\rm RM}_{\pi^+}$ by a factor of 2.3 to bring it to $H^{\rm RM}_{\rm ch}$ that agrees with $H^{\rm KLN}_{\rm ch}$ at $W = 130$ GeV, then the extrapolation to LHC energy results in

$$H^{\rm RM}_{\rm ch}(5500) = 9.03, \qquad R^{\rm KLN}_{\rm ch}(5500) = 10.90\ . \tag{31}$$

The uncertainty of the species differences in charged particles at 5500 GeV renders the above numbers to be essentially comparable, i.e., $\sim 10 \pm 1$.

5 Discussion

Although the consideration in the preceding section leads to a result, expressed in Eq. (31), that does not show any drastic difference between RM and KLN, there are many issues opened up in need of better understanding. We are unable to give any physical interpretation for the value of γ in Eq. (25). If a gluon converts to a linear combination of $q\bar{q}$ pairs, γ should be 1. Thus our lack of understanding of LPHD is replaced by that of Eq. (24), which relates partons to recombining quarks, instead of relating partons to hadrons.

Since Eq. (24)-(26) are based on phenomenology at RHIC energies, one can question the accuracy of that analysis. The data on $J(p_T, W)$ are fitted by exponential formula for a limited p_T range in Eq. (13)-(15). There are two flaws in that analysis. The data points are not strictly exponential, since the spectra become power-law behaved at higher p_T. Thus the parameters C and T depend on what p_T range is used in the fitting procedure. We have used the lowest possible region $0.25 < p_T < 1.0$ GeV/c for $W = 130$ and 200 GeV (not yet available for 62.4 GeV), since no data points exist for $p_T < 0.25$ GeV/c. However, we know that hadronization at $p_T < 0.25$ GeV/c is complicated, since soft gluon radiation that mutates the colors of the recombining quarks can contribute to a remnant pool of very soft partons that might build up unknown hadron multiplicities and can be the manifestation of a major part of the saturation phenomenon. Furthermore, the question of how entropy can increase from the initial to the final state has not been carefully investigated.

We have discussed the hadronization problem of pion production. The production of other mesons and baryons can similarly be considered [14], but at low p_T the mass effect as well as resonance decay become important, and a simple formula for $N_{\rm ch}/N_q$ is unreliable, especially when W is extrapolated to LHC energy. In other words, even if N_π/N_q can be calculated, the ratio $N_{\rm ch}/N_\pi$ may have a W dependence that can invalidate any naive extrapolation. One can restrict the attention to π^+ production only when the data become available at LHC, but then how is LPHD modified by such a restriction?

Finally, since it is expected that high-p_T jets will be copiously produced at LHC, the fragmentation of those jets to low-p_T hadrons will undoubtedly introduce a new component to the hadron multiplicity that we have thus far ignored. Such particles are not constrained by saturation physics, so the prediction based on the extrapolation of Eq. (21) may very likely be inadequate.

These various issues may undermine the meaningfulness of Eq. (31). While they are issues to be investigated in due time, the study in this paper shows at least that in the narrow context of hadronization by recombination our result is not incompatible with the simple application of LPHD.

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