

# Autocorrelation of Hadrons in Jets Produced in Heavy-Ion Collisions

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## Abstract

Autocorrelation of two pions produced in heavy-ion collisions at intermediate  $p_T$  is calculated in the framework of the recombination model. The differences of the pseudo-rapidities and azimuthal angles of the two pions are related to the angle between two shower partons in a jet. It is shown how the autocorrelation distribution reveals the properties of jet cone of the shower partons created by high- $p_T$  partons in hard collisions.

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# 1 Introduction

Correlations among hadrons produced at high and intermediate  $p_T$  in heavy-ion collisions (HIC) have generated considerable interest in their implications on how the hadronization mechanism and jet structure may differ from those in  $pp$  collisions [1]-[4]. Experimental investigations in the subjects can broadly be divided into two types: those that use triggers to identify near- and away-side jets [1, 2, 4, 5] and those that use no triggers [3, 4, 6, 7]. Theoretical studies are even more varied in their approaches [8]-[12]. The application of the notion of autocorrelation that is used extensively in time-series analysis to multiparticle production in HIC was pioneered by Trainor and his collaborators [3, 4, 13, 14], and has generated a wealth of information independent of triggers and their biases [6, 7]. Theoretical interpretation of autocorrelation has been slow in its development. In this paper we present the first prediction of how autocorrelation should behave at intermediate  $p_T$  in the framework of parton recombination [10, 15, 16].

The use of trigger has its advantages, especially in showing the properties of the away-side azimuthal distribution that reveals the effects of jet quenching. However, it is necessary to subtract the background, which is not unambiguous. Autocorrelation is a measure of the difference between two nearby values of a variable, with all other variables being integrated over. When those values are close, they are dominated by contributions arising from the same jet in an event. No background subtraction is needed. So far autocorrelation in the data from the Relativistic Heavy-Ion Collider (RHIC) has been analyzed for differences in pseudorapidity  $\eta$  and in azimuthal angle  $\phi$ , but only at low  $p_T$  [7, 13]. The model that we shall use to study the autocorrelation in jets involve thermal-shower recombination for which the

reliable  $p_T$  region is above 2 GeV/c. Thus at this point our predictions cannot be compared to the results of the autocorrelation analysis of the experimental data. Nevertheless, on theoretical grounds it is of interest to show how the angular distribution of shower partons can be related to the autocorrelation of pions in the differences in  $\eta$  and  $\phi$ . We await with anticipation the relevant data that are forthcoming.

## 2 The Problem

Autocorrelation is a measure of two-particle correlation in multiparticle production with a minimum loss of information and without trigger bias. It has been studied extensively in  $pp$  and HIC [6, 7, 13]. At intermediate to high  $p_T$  the dominant contribution to the two-particle correlation comes from jets, when the momenta for the two particles are close together. Our problem in this paper is to calculate the autocorrelation distribution from a model in which the shower partons in a jet has certain prescribed properties, and in this section we outline our plan of attack, leaving the details to the following sections.

Let  $x_i$  be an attribute of the momentum  $\vec{p}_i$  of the  $i$  th particle, such as its angle relative to some axis. The two-particle correlation function in terms of  $x_i$  is

$$C_2(x_1, x_2) = \rho_2(x_1, x_2) - \rho_1(x_1)\rho_1(x_2) \quad (1)$$

where  $\rho_1$  and  $\rho_2$  are one- and two-particle distributions, respectively. For autocorrelation one defines the sum and difference of  $x_i$

$$x_{\pm} = x_2 \pm x_1 \quad (2)$$

and rewrite Eq. (1) in the form

$$C_2(x_+, x_-) = \rho_2(x_+, x_-) - \rho_1((x_+ - x_-)/2)\rho_1((x_+ + x_-)/2) . \quad (3)$$

This would vanish if there is no correlation in  $\rho_2$ . Anticipating correlation in the variable  $x_-$  and mild dependence on  $x_+$ , one defines the autocorrelation distribution to be

$$A(x_-) = \frac{1}{R} \int_R dx_+ C_2(x_+, x_-) , \quad (4)$$

where the integration is carried out over a range  $R$ . Usually, if there are other variables in the problem, which are, however, not germane to the correlation measure, they are integrated over also. If the range  $R$  is wide enough so that the boundaries depend on  $x_-$ , then one must proceed carefully to account for the  $x_-$  dependence arising from  $R$ . Details of that problem related to binning can be found in [13, 17]. In our consideration in the following  $R$  will be small enough not to involve such complications. Clearly, only the correlated part in  $\rho_2(x_+, x_-)$  contributes to  $A(x_-)$ . If  $\rho_2(x_+, x_-)$  has mild dependence on  $x_+$ , not much information is lost by the integration in Eq. (4).  $A(x_-)$  treats the two particles on equal footing and requires no subtraction of background besides what is explicit in Eq. (3).

In HIC if the transverse components of  $\vec{p}_i$  are  $> 2$  GeV/c, then jets are involved; furthermore, if  $\vec{p}_1$  and  $\vec{p}_2$  are nearly collinear, then the two particles are highly likely to be the particles in the same jet. Thus the angular differences between the momentum vectors provide information about the structure of the jet. Let the angular variables of  $\vec{p}_1$  and  $\vec{p}_2$ , referred to the longitudinal axis, be  $(\theta_1, \phi_1)$  and  $(\theta_2, \phi_2)$ , respectively. Define

$$\theta_{\pm} = \theta_2 \pm \theta_1, \quad \phi_{\pm} = \phi_2 \pm \phi_1. \quad (5)$$

In a central collision  $\phi_+$  is irrelevant by azimuthal symmetry, which we shall assume. Thus the

essential variables for correlation are  $\theta_+$ ,  $\theta_-$  and  $\phi_-$ . If the correlation function is determined experimentally at midrapidity with a narrow rapidity window, the range of  $\theta_+$  is not large. Then in applying Eq. (4) to this problem, we have

$$A(\theta_-, \phi_-) = \frac{1}{R_{\theta_+}} \int_{R_{\theta_+}} d\theta_+ C_2(\theta_+, \theta_-, \phi_-) , \quad (6)$$

where  $R_{\theta_+}$  is some range in  $\theta_+$  that is not constrained by the ranges of  $\theta_-$  and  $\phi_-$  of interest. Experimentally, the  $p_T$  variables are integrated over specific ranges of choice, and are not expressed explicitly. It is important to note that  $A(\theta_-, \phi_-)$  depends only on the difference angular variables, and is therefore, in principle, independent of the coordinate system in which the angles are defined. The only angle associated with two momentum vectors that is independent of the coordinate system is the angle  $\chi$  between the two vectors, i.e.,

$$\cos \chi = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\phi_2 - \phi_1) , \quad (7)$$

which can be expressed in terms of  $\theta_{\pm}$  and  $\phi_-$  as

$$\cos \chi = \frac{1}{2} [\cos \theta_- (1 + \cos \phi_-) + \cos \theta_+ (1 - \cos \phi_-)] . \quad (8)$$

How this is used in Eq. (6) to obtain the autocorrelation distribution will be discussed in Sec. 4.

The angle  $\chi$  provides the crucial link between the observables and the variables that can suitably describe the dynamics of the partons that are associated with jets. At intermediate and high  $p_T$  in HIC the partons that hadronize are the shower partons in jets produced by hard scattering. The axis in reference to which those partons can best be described is the jet axis. For detection at midrapidity the jet directions are approximately perpendicular to the beam axis. Thus to relate the angular variables of the partons, referred to the jet axis, to

the angular variables of the hadrons, referred to the beam axis, is a complicated geometrical problem. In the recombination model the hadrons and the constituent partons are collinear. The angle between two shower partons is therefore also the angle between the two pions that they hadronize into. That angle is  $\chi$ . Hence,  $\chi$  serves as the bridge that connects the momentum space of the partons in the underlying dynamics to the momentum space of the hadrons that can be measured. Autocorrelation in terms of  $\theta_-$  and  $\phi_-$  can thus directly reveal the angular properties of the shower partons via  $\chi$ . To put this strategy into concrete formulation, we proceed first to the momentum space of the partons in the next section, and then to the momentum space of the hadrons in the following section.

### 3 Correlation between Shower Partons

Suppose that in a HIC a hard scattering takes place that sends a parton to a momentum  $\vec{k}$  with a large transverse momentum. Such a hard parton generates a shower of partons whose momentum-fraction distributions have been determined in Ref. [18]. Those semi-hard shower partons can recombine with the soft thermal partons to form pions at intermediate  $p_T$ . To study the correlation between two pions, both in the intermediate  $p_T$  range, we need to consider two shower and two thermal partons in the combination  $(\mathcal{ST})(\mathcal{ST})$ , where  $\mathcal{S}$  denotes the shower-parton distribution (SPD) and  $\mathcal{T}$  the thermal parton distribution [8, 10, 16]; the parentheses in  $(\mathcal{ST})$  imply that the partons bracketed are to recombine. Since only collinear partons can recombine, the angle  $\chi$  between the two pions that are formed is the same as the angle between the two shower partons, whose momenta magnitudes are generally larger than the two softer thermal partons. Let us then concentrate on the two shower partons and

consider the joint distribution

$$\{\mathcal{SS}\}^{jj'}(\vec{q}_1, \vec{q}_2) = \xi \sum_i \int dk k f_i(k) \left\{ S_i^j \left( \frac{q_1}{k}, \psi_1, \beta_1 \right), S_i^{j'} \left( \frac{q_2}{k - q_1}, \psi_2, \beta_2 \right) \right\}, \quad (9)$$

which is in the form that has been used previously in Ref. [10]. The curly brackets denote symmetrization of the momentum fractions of the two shower partons [8], but it is a process that is not important in the following, since our emphasis will be on the angular variables. For the same reason, we shall not be concerned with  $\xi$ , which denotes the fraction of hard partons that emerge from the dense medium to hadronize, and the integral over  $k$ , weighted by  $f_i(k)$  that is the distribution of the hard parton  $i$  in HIC.  $\psi_1$  and  $\psi_2$  are the polar angles of the  $j$  and  $j'$  shower parton momenta,  $\vec{q}_1$  and  $\vec{q}_2$ , with reference to the jet axis,  $\hat{k}$ ;  $\beta_1$  and  $\beta_2$  are their corresponding azimuthal angles.  $S_i^{j,j'}$  are the SPDs.

The angle between  $\vec{q}_1$  and  $\vec{q}_2$ , denoted by  $\chi$  also, can be expressed similarly as in Eq. (7)

$$\cos \chi = \cos \psi_1 \cos \psi_2 + \sin \psi_1 \sin \psi_2 \cos(\beta_2 - \beta_1). \quad (10)$$

If we define, as before,

$$\psi_{\pm} = \psi_2 \pm \psi_1, \quad \beta_{-} = \beta_2 - \beta_1, \quad (11)$$

then we have the alternate form similar to Eq. (8), which in turn can be written as

$$\cos \chi = A + B \cos \beta, \quad (12)$$

where

$$A = \frac{1}{2}(\cos \psi_{-} + \cos \psi_{+}), \quad (13)$$

$$B = \frac{1}{2}(\cos \psi_{-} - \cos \psi_{+}), \quad (14)$$

and  $\beta = \beta_-$  for brevity. Note that  $A$  and  $B$  are linear in  $\cos \psi_{\pm}$ , while the corresponding terms in Eq. (10) are quadratic in  $\cos \psi_{1,2}$ . Since  $\chi$  serves as the bridge to the hadron momentum space, we want to determine here the distribution in  $\chi$  that correspond to the dynamical properties of the jet cone defined with respect to  $\vec{k}$ .

The joint shower-parton distribution, as expressed in Eq. (9), is based on the assumption that the shower partons are dynamically independent, but kinematically constrained by momentum conservation

$$q_1 + q_2 \leq k. \quad (15)$$

The constraint on the momentum magnitudes in Eq. (15) does not imply angular constraint on  $\psi_1$  and  $\psi_2$ . We shall assume angular independence of the two partons, which implies that their distribution  $G_2(\psi_1, \psi_2)$  is factorizable

$$G_2(\psi_1, \psi_2) = G_1(\psi_1)G_1(\psi_2), \quad (16)$$

where  $G_1(\psi)$  is the single-parton angular distribution in a jet cone. We assume that it has a Gaussian form

$$G_1(\psi) = \exp\left(-\psi^2/2\sigma^2\right) \quad (17)$$

with a width  $\sigma$  that is basically unknown, since no reliable theory is calculable at intermediate  $k$ . We make the assumption here that  $\sigma$  is independent of the momentum fraction of the shower parton in a jet. That assumption renders significant simplification of our consideration below, and is a reasonable first step in this exploratory study of autocorrelation. If we can relate  $\sigma$  to some observable through autocorrelation, we will have achieved the objective of probing the microscopic dynamics by phenomenology, at least in the first approximation.



Given the two-parton angular distribution  $G_2(\psi_1, \psi_2)$  shown in Eqs. (16) and (17) we can calculate the  $\chi$  distribution by allowing for all possible orientations of the two vectors  $\vec{q}_1$  and  $\vec{q}_2$ . Denoting it by  $H(\chi)$ , we have

$$H(\chi) = \int d\cos\psi_2 d\cos\psi_1 d\beta G_2(\psi_1, \psi_2) \delta[\cos\chi - (A + B\cos\beta)], \quad (18)$$

where Eq. (12) has been used to constrain the four angles  $\psi_{\pm}, \beta$  and  $\chi$ . Since  $H(\chi)$  is invariant under the interchange of  $\vec{q}_1$  and  $\vec{q}_2$ , we consider only the ranges

$$0 \leq \psi_1 \leq \psi_2 \leq \alpha. \quad (19)$$

In actual calculation we set  $\alpha = \pi/4$ . In view of the definitions of  $A$  and  $B$  given in Eqs. (13) and (14), we change the integration variables to  $\psi_{\pm}$  and  $\beta$ . After some algebra we obtain

$$H(\chi) = \frac{1}{4} \int_{\chi}^{2\alpha} d\psi_+ \int_0^{\psi_m} d\psi_- \frac{(\cos\psi_- - \cos\psi_+)g(\psi_+, \psi_-)}{[(\cos\psi_- - \cos\chi)(\cos\chi - \cos\psi_+)]^{1/2}} \quad (20)$$

where

$$\psi_m = \min(\chi, 2\alpha - \psi_+), \quad (21)$$

and

$$g(\psi_+, \psi_-) = \exp\left(-\frac{\psi_+^2 + \psi_-^2}{4\sigma^2}\right). \quad (22)$$

In Fig. 1 (a) we show  $H(\chi)$  for four values of  $\sigma$ . To see more clearly the dependence of the width of  $H(\chi)$  on the width  $\sigma$  of  $G_1(\psi)$ , we normalize the function  $H(\chi)$  by its value at  $\chi = 0$ , by defining

$$\tilde{H}(\chi) = \frac{H(\chi)}{H(0)}. \quad (23)$$

It can be shown that in the approximation  $\sigma \ll \alpha$ ,  $H(\chi)$  has the limit, as  $\chi \rightarrow 0$ ,  $H(0) \approx (\pi/4)\sigma^2 \exp(-\sigma^2/4)$ . The corresponding normalized distribution  $\tilde{H}(\chi)$  is shown in Fig. 1 (b). In the  $\sigma$  range illustrated, the width of  $\tilde{H}(\chi)$  is very closely given by  $\sigma_H = \sqrt{2}\sigma$ .

It is important to recognize that the factorizable form of  $G_2(\psi_1, \psi_2)$  in Eq. (16) does not imply an absence of correlation between  $\psi_1$  and  $\psi_2$ . That is because the two shower partons with momenta  $\vec{q}_1$  and  $\vec{q}_2$  are from the same jet, and the angles  $\psi_1$  and  $\psi_2$  refer to the same jet axis along  $\vec{k}$ . When  $G_2(\psi_1, \psi_2)$  is included in the expression for  $\{\mathcal{SS}\}^{jj'}(\vec{q}_1, \vec{q}_2)$  in Eq. (9), the contribution to the two-parton distribution  $\rho_2(\vec{q}_1, \vec{q}_2)$  is not factorizable. On the other hand,  $\rho_2(\vec{q}_1, \vec{q}_2)$  contains a factorizable part  $\rho_1(\vec{q}_1)\rho_1(\vec{q}_2)$ , in which the two shower partons belong to two independent jets that are randomly related to each other. Such a component would be cancelled in the calculation of the correlation function  $C_2(\vec{q}_1, \vec{q}_2)$  that can be defined as in Eq. (1) for partons.

## 4 Autocorrelation between Hadrons

Having obtained the distribution in  $\chi$ , the angle between two shower partons in a jet, we can now determine the correlation between two hadrons in a jet. We shall consider only the pions, since they are the dominant hadrons in a jet. As we have stated earlier, we consider the intermediate  $p_T$  region where the thermal-shower recombination is most important, i.e.,  $3 < p_T < 8$  GeV/c in central Au+Au collisions [8, 15]. Furthermore, we restrict our attention here to only the pions produced at midrapidity so that all momentum vectors are nearly transverse to the beam direction. In that case we can simplify our notation by denoting the pion transverse momentum  $p_T$  and the parton transverse momentum  $q_T$  by  $p$  and  $q$ ,

respectively. The  $(\mathcal{TS})(\mathcal{TS})$  contribution to the two-particle distribution is then

$$\frac{dN_{\pi\pi}^{TSTS}}{p_1 dp_1 d\eta_1 d\phi_1 p_2 dp_2 d\eta_2 d\phi_2} = \frac{1}{(p_1 p_2)^3} \int dq_1 dq_2 d\cos\psi_1 d\cos\psi_2 d\beta_1 d\beta_2 \mathcal{T}(p_1 - q_1) \mathcal{T}(p_2 - q_2) \{\mathcal{SS}\}(q_1, \psi_1, \beta_1; q_2, \psi_2, \beta_2) \Delta(\psi_1, \beta_1, \eta_1, \phi_1; \psi_2, \beta_2, \eta_2, \phi_2), \quad (24)$$

where  $\mathcal{T}$  is the thermal distribution of exponential form [15], and  $\{\mathcal{SS}\}$  is given in Eq. (9). The recombination functions, written out explicitly in Ref.[10], have already been integrated over, resulting in (a) the momenta of the thermal partons being  $p_i - q_i$ , and (b) a constraint between the pions' angular variables  $\theta_i, \phi_i$  and the shower partons' angular variables  $\psi_i, \beta_i$ , contained in the  $\Delta$  function in Eq. (24). We have exhibited only the essential content of the  $(\mathcal{TS})(\mathcal{TS})$  recombination, where the momenta of the thermal and shower partons that recombine are collinear, so all the dynamical characteristics of the problem are in  $\{\mathcal{SS}\}$ , which we have studied in the preceding section, while all the kinematical relationships to the observed pions are in  $\Delta$ . Although the pseudorapidity variables  $\eta_1$  and  $\eta_2$  appear in Eq. (24), it is only a technical step to be taken later to relate their difference to the angular differences,  $\theta_-$  and  $\phi_-$ , that appear in the autocorrelation, Eq. (6). Our task now is to first relate  $A(\theta_-, \phi_-)$  to the two-particle distribution in Eq. (24).

In our study of the autocorrelation our emphasis has been on the dependence on the angular differences,  $\theta_-$  and  $\phi_-$ , relegating the  $p_T$  values to the category of other variables that are to be integrated over. If we define the full correlation function to be

$$C_2(1, 2) = \rho_2(1, 2) - \rho_1(1)\rho_1(2), \quad (25)$$

where the arguments symbolize the whole sets of variables of the two detected particles, then the LHS of Eq. (24) represents the  $TSTS$  component of  $\rho_2(1, 2)$ , which we now denote

by  $\rho_2^{TSTS}(1, 2)$ . Both in the analysis of the experimental data and in our construction of autocorrelation, the  $p_T$  values are integrated over some chosen ranges. In our work here the autocorrelation does not depend sensitively on that  $p_T$  range so long as it is the intermediate  $p_T$  range where  $TS$  recombination is dominant for the formation of a pion. Let us then define the integrated correlation

$$\Gamma(\eta_1, \phi_1, \eta_2, \phi_2) = \int dp_1 dp_2 p_1 p_2 C_2(1, 2). \quad (26)$$

With  $\eta_{\pm}$  defined by

$$\eta_{\pm} = \eta_2 \pm \eta_1 \quad (27)$$

and  $\phi_{\pm}$  given in Eq. (5),  $\Gamma(\eta_{\pm}, \phi_{\pm})$  receives its contribution mainly from the  $TSTS$  component,  $\rho_2^{TSTS}(1, 2)$ , expressed in Eq. (24), when  $\eta_-$  and  $\phi_-$  become small.

We now focus on  $\rho_2^{TSTS}(1, 2)$  and, in particular, on the  $\Delta$  function in Eq. (24), which contains only the angular variables of the parton and pion momenta. As we have emphasized earlier, with the autocorrelation  $A(\theta_-, \phi_-)$  as the aim of our analysis, the angle  $\chi$  is the bridge between the parton and pion angular variables that are independent of the coordinate system.

With that simplification in mind we write  $\Delta$  in the form

$$\Delta(\psi_1, \beta_1, \eta_1, \phi_1; \psi_2, \beta_2, \eta_2, \phi_2) = \int d\cos\chi \delta[\cos\chi - (A + B\cos\beta_-)] \delta[\cos\chi - (C + D\cos\phi_-)], \quad (28)$$

where  $A$  and  $B$  are functions of  $\psi_{\pm}$ , defined in Eqs. (13) and (14), while  $C$  and  $D$  are similar functions of  $\theta_{\pm}$  that follow from Eq. (8), and are explicitly

$$C = \frac{1}{2}(\cos\theta_- + \cos\theta_+), \quad (29)$$

$$D = \frac{1}{2}(\cos\theta_- - \cos\theta_+). \quad (30)$$

It is clear that the two  $\delta$  functions in Eq. (28) express the two ways of referring  $\chi$  to the two different sets of angles defined with respect to the jet axis on the one hand, and to the beam axis on the other.

Substituting Eqs. (9) and (28) into Eq. (24), and performing the angular integration as in Eq. (18), we obtain  $H(\chi)$  multiplied by some factors involving the momenta magnitudes. Such factors are irrelevant to our study, so we normalize it out by defining

$$\tilde{C}_2(\eta_{\pm}, \phi_{\pm}) = \frac{\Gamma(\eta_+, \eta_-; \phi_+, \phi_-)}{\Gamma(\eta_+, 0; \phi_+, 0)}. \quad (31)$$

Pending the connection between  $\eta_{\pm}$  and  $\theta_{\pm}$ , we have from the above process of integrations

$$\tilde{C}_2(\theta_{\pm}, \phi_{\pm}) = \int d\cos\chi \tilde{H}(\chi) \delta[\cos\chi - (C + D\cos\phi_-)]. \quad (32)$$

This integration over  $\chi$  need not be performed explicitly, since it is only necessary to determine the domain of  $\theta_{\pm}$  and  $\phi_-$  that corresponds to each fixed value of  $\chi$  and denote the result as  $\tilde{H}(\theta_{\pm}, \phi_-)$ . The final step is to integrate over  $\theta_+$  as in Eq. (6). Conceptually, it is simpler to visualize the average angle  $\bar{\theta}$ , defined by

$$\bar{\theta} = \theta_+/2, \quad (33)$$

which should be around  $\pi/2$ , since the two detected pions are restricted to a narrow interval at mid-rapidity, corresponding to a jet at roughly  $\pi/2$  relative to the beam axis. If we denote the interval of  $\bar{\theta}$  by  $2\epsilon$ , then we have

$$\tilde{A}(\theta_-, \phi_-) = \frac{A(\theta_-, \phi_-)}{A(0, 0)} = \frac{1}{2\epsilon} \int_{\pi/2-\epsilon}^{\pi/2+\epsilon} d\bar{\theta} \tilde{C}_2(\theta_+, \theta_-, \phi_-), \quad (34)$$

while  $\phi_+$  is an irrelevant angle in an azimuthally symmetric problem. Experimentally,  $\tilde{A}(\theta_-, \phi_-)$  is to be determined by use of Eq. (6).

The transference from  $\theta_-$  to  $\eta_-$  involves  $\theta_+$  and is just an algebraic problem. Since

$$\eta_- = \ln \left( \frac{\tan \theta_1/2}{\tan \theta_2/2} \right), \quad (35)$$

$$\theta_1 = \frac{1}{2}(\theta_+ - \theta_-), \quad \theta_2 = \frac{1}{2}(\theta_+ + \theta_-), \quad (36)$$

we can solve for  $\theta_-$  in terms of  $\eta_-$  and  $\theta_+$ . For  $\bar{\theta}$  not far from  $\pi/2$ ,  $\theta_-$  behaves essentially like  $\eta_-$ . Substituting the exact dependence of  $\theta_-$  on  $\eta_-$  and  $\theta_+$  into  $\tilde{C}_2(\theta_+, \theta_-, \phi_-)$  in Eq. (34) and then average over  $\bar{\theta}$ , we obtain

$$\tilde{A}(\eta_-, \phi_-) = \frac{1}{2\epsilon} \int_{\pi/2-\epsilon}^{\pi/2+\epsilon} d\bar{\theta} \tilde{H}(\eta_-, \phi_-, \bar{\theta}). \quad (37)$$

This is our main result, except for the 3D display of  $\tilde{A}(\eta_-, \phi_-)$  after numerical computation. The essence of autocorrelation is basically already contained in Fig. 1 (b), where  $\tilde{H}(\chi)$  is shown for various values of the jet cone width  $\sigma$ . For every value of  $\chi$  there is a set of values of  $\eta_-$ ,  $\phi_-$  and  $\bar{\theta}$ , whose contour plot gives a representation of  $\tilde{A}(\eta_-, \phi_-)$ .

In Fig. 2 we plot  $\tilde{A}(\eta_-, \phi_-)$  for  $\sigma = 0.2$ . There is very little sensitivity on the dependence on  $\epsilon$  in Eq. (37); we set it at  $\epsilon = 0.3$ . The dependence on  $\sigma$  is, however, significant, as we have already seen in Fig. 1. The general shape of  $\tilde{A}(\eta_-, \phi_-)$  is similar, the peak being broader for higher values of  $\sigma$ . The utility of this result is, of course, the reverse. When the data on  $A(\eta_-, \phi_-)$  become available, we can use  $A(\eta_-, \phi_-)$  to infer what the corresponding cone width  $\sigma$  should be. To facilitate that deduction, we show in Fig. 3 (a)  $\tilde{A}(\eta_-, 0)$  and (b)  $\tilde{A}(0, \phi_-)$  for various values of  $\sigma$ . We see that the width in  $\eta_-$  is numerically larger than that in  $\phi_-$ . While in the small  $\eta_-$  region one has  $\eta_- \sim \phi_-$ , the two variables should not be directly compared since  $\eta_-$  is not an angle. The more important implication of this result is that we have in Fig. 3 peaks in the measurable variables  $\eta_-$  and  $\phi_-$  for four values

of the theoretical variable  $\sigma$  whose magnitude is not known from first principles. Thus any experimental information on the autocorrelation can give us direct information on the nature of the jet cone.

## 5 Conclusion

We have determined the autocorrelation distribution in  $\eta_-$  and  $\phi_-$  for two pions produced in HIC in the intermediate  $p_T$  region. We have used the parton recombination model to relate the hadronic angles to the partonic angles. By emphasizing thermal-shower parton recombination, we have exploited the equivalence of the angle between the two shower partons and that between the two observed pions. Thus when the data on autocorrelation at intermediate  $p_T$  become available, the width of the observed peak can then be related to the width of the jet cone, which is a property of the jet physics in HIC that is the goal of this study.

It should be mentioned that our investigation has been focused on the angular relationship among the partonic and hadronic momenta. That relationship is independent of  $p_T$  so long as all the transverse momenta in the problem are in the intermediate  $p_T$  region. Since the thermal partons are soft, the shower parton and the pion that is formed by  $TS$  recombination are roughly of the same magnitudes and are collinear. Thus the angle  $\chi$  between two shower partons in a jet is the same as the angle  $\chi$  between the two pions, independent of their momentum magnitudes. However, we have made use of the simplifying assumption that the jet cone has a width  $\sigma$  that is independent of the momentum fraction of the shower partons. While it is sensible to make that assumption in this first attempt to calculate the

autocorrelation, we expect the realistic situation to be more complicated. Once that dependence on momentum fraction is considered, the magnitude of the hard parton momentum becomes relevant, and the final result on autocorrelation will exhibit dependence on the  $p_T$  range, even within the intermediate  $p_T$  region  $3 < p_T < 8$  GeV/c. Needless to add, when  $p_T$  goes outside that region  $TS$  recombination no longer dominates and the basis for our study in this paper will have to be revised.

Despite the simplifying assumption made in this paper, it will be of great interest to compare our result with the forthcoming data on autocorrelation. As far as we know, there exists no other theoretical study in the subject that relates the observables to the partonic structure in a jet, since no other hadronization scheme has been shown to be reliable in the  $p_T$  range considered. In addition to pions one can also study proton and other heavier particles in the jets and their autocorrelations. Even in the pion sector alone it is of interest to consider the complications arising from different charge states, since multiple shower partons in a jet can exhibit dependence on their flavors. Thus there remains much to be learned in the subject, in which the present study is only a beginning.

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## Figure Captions

Fig. 1. (a) Distributions in the angle  $\chi$  for four values of the width parameter  $\sigma$  of the jet cone; (b) normalized distributions  $\tilde{H}(\chi)$ .

Fig. 2. Normalized autocorrelation function  $\tilde{A}(\eta_-, \phi_-)$  in 3D plot in  $\eta_-$ - $\phi_-$  for  $\sigma = 0.2$ .

Fig. 3. (a) Normalized autocorrelation  $\tilde{A}(\eta_-, 0)$  for four values of  $\sigma$ ; (b) normalized  $\tilde{A}(0, \phi_-)$ .