Learning and Monetary Policy

Lecture 3 – Optimal Monetary Policy and Learning in the New Keynesian Model

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INTRODUCTION

This lecture reviews the results of Evans & Honkapohja (REStud, 2003; JMCB, 2003, ScandJE, 2006) and Evans & McGough (JMCB, 2007).

- We start from the standard "new Phillips curve/IS curve" NK model,

- optimal monetary policy under RE and we look at two potential problems:
  - Indeterminacy (multiple equilibria) and instability under learning

- We find: a well chosen "expectations based" i_t rule is superior to purely "fundamentals based" rules.

OUTLINE

- Optimal monetary policy under RE in the NK model.

- Problems of indeterminacy and instability under learning with fundamentals-based i_t rules.

- Desirability of "expectations based" rules

- Extensions: (i) discussion of approximate targeting rules
  (ii) structural parameter learning
  (iii) robustness to structural parameter uncertainty.

MACRO MODEL

The structural model is:

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t \tag{IS}$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{PC}$$

$x_t =$ "output gap" and $\pi_t =$ inflation rate,

g_t, u_t are observable with

$g_t = \mu g_{t-1} + \tilde{g}_t$ and $u_t = \rho u_{t-1} + \tilde{u}_t$.

See e.g. Woodford (various) and "The Science of Monetary Policy," Clarida, Gali & Gertler (JEL, 1999)

OPTIMAL MONETARY POLICY WITH COMMITMENT UNDER RE

To complete the model we add a policy rule for i_t.

The policy maker aims to minimize

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\alpha x_{t+s}^2 + \pi_{t+s}^2\right).$$

Note: x target of 0 (no inflation bias), π target of 0 (for simplicity).

Distinguish between policy with and without commitment.

We will focus on the <u>commitment</u> case. EH (REStud, 2003 examine discretion)

Gains From Commitment

Two possible sources of gain from commitment

- Classic Inflation bias. For the objective function

$$\alpha(x_{t+s} - \bar{x})^2 + \pi_{t+s}^2, \text{ where } \bar{x} > 0$$

  policy makers gain by committing to a policy with $Ex_{t+s} = 0$ (Kydland-Prescott). This is <u>not</u> our focus here: we set $\bar{x} = 0$.

- Stabilization bias. Arises from forward looking PC curve. Commitment gains arise even with $\bar{x} = 0$.

OPTIMAL POLICY WITH COMMITMENT

From the FOCs we obtain

$$\begin{aligned}\lambda\pi_t &= -\alpha x_t \\ \lambda\pi_{t+s} &= -\alpha(x_{t+s} - x_{t+s-1}), \text{ for } s = 1, 2, \ldots\end{aligned}$$

- Optimal discretionary policy is $\lambda\pi_t = -\alpha x_t$, all t

- Optimal policy with commitment is time inconsistent

- We adopt the timeless perspective optimal policy (see Woodford and McCallum/Nelson),

  $$\lambda\pi_t = -\alpha(x_t - x_{t-1}), \text{ all } t, \qquad \text{(OPT)}$$

  i.e. follow same rule in first period too.

OPTIMAL SOLUTION UNDER RE

Combining PC and OPT $\longrightarrow$ optimal REE

$$
\begin{aligned}
x_t &= \bar{b}_x x_{t-1} + \bar{c}_x u_t, \\
\pi_t &= \bar{b}_\pi x_{t-1} + \bar{c}_\pi u_t.
\end{aligned}
$$

where $\bar{b}_x$ is the root $0 < \bar{b}_x < 1$ of

$$\beta \bar{b}_x^2 - \gamma \bar{b}_x + 1 = 0,$$

and $\gamma = 1 + \beta + \lambda^2/\alpha$.

We still need an <u>interest rate reaction function</u> that implements the optimal REE.

FUNDAMENTALS FORM OF OPTIMAL i_t RULE

- Compute x_t, $E_t\pi_{t+1}$ and $E_t x_{t+1}$ for optimal REE.

- Insert into IS curve to get the "fundamentals-based" optimal i_t-rule

$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t,$$

  where ψ_i depend on $\lambda, \alpha, \rho, \varphi, \beta$.

This i_t rule is consistent with the optimal REE. But

- Will it lead to "determinacy"?

- Will it lead to stability under learning?

DETERMINACY

Combining IS, PC and the fundamentals-based i_t rule gives the reduced form

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} n_{11} & 0 \\ n_{21} & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & p_{12} \\ 0 & p_{22} \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix},$$

$$y_t = M E_t^* y_{t+1} + N y_{t-1} + P v_t.$$

Recall that we say a model is "determinate" if a unique (nonexplosive) solution, and "indeterminate" if there are multiple solutions. These will include stationary sunspot solutions.

Compare eigenvalues of the matrix of the stacked first-order system to the number of predetermined variable.

DETERMINACY RESULTS: FUNDAMENTALS BASED REACTION FUNCTION

Proposition 1: *Under the fundamentals based reaction function there are parameter regions in which the model is determinate and other parameter regions in which it is indeterminate.*

Calibrations

W: $\beta = 0.99$, $\varphi = (0.157)^{-1}$, $\lambda = 0.024$.
CGG: $\beta = 0.99$, $\varphi = 4$, $\lambda = 0.075$
MN: $\beta = 0.99$, $\varphi = 0.164$, $\lambda = 0.3$.

Indeterminate for output policy weights $\alpha < \hat{\alpha}$, where
$\hat{\alpha} = 0.16$ (W), 7.5 (CGG), 277 (MN)
Hence in some cases this i_t rule is also consistent with inefficient REE.

Review: Learning and E-stability

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t.$$

The optimal REE takes the form

$$y_t = \bar{a} + \bar{b}y_{t-1} + \bar{c}v_t,$$

and the corresponding expectations are

$$\begin{aligned} E_t y_{t+1} &= \bar{a} + \bar{b}y_t + \bar{c}E_t v_{t+1} \\ &= \bar{a} + \bar{b}(\bar{a} + \bar{b}y_{t-1} + \bar{c}v_t) + \bar{c}Fv_t \end{aligned}$$

Replace by LS learning:

$$E_t^* y_{t+1} = a_t + b_t(a_t + b_t y_{t-1} + c_t v_t) + c_t F v_t,$$

with a_t, b_t, c_t updated using LS. The question: over time does

$$(a_t, b_t, c_t) \rightarrow (\bar{a}, \bar{b}, \bar{c})?$$

E-STABILITY

Stability under learning is analyzed using E-stability. Reduced form:

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t.$$

For the PLM (Perceived Law of Motion)

$$y_t = a + by_{t-1} + cv_t.$$

$$E_t^* y_{t+1} = (I + b)a + b^2 y_{t-1} + (bc + cF)v_t.$$

This $\longrightarrow$ ALM (Actual Law of Motion)

$$y_t = M(I + b)a + (Mb^2 + N)y_{t-1} + (Mbc + NcF + P)v_t.$$

Mapping from PLM to ALM

$$T(a, b, c) = (M(I + b)a, Mb^2 + N, Mbc + NcF + P).$$

T: PLM $\longrightarrow$ ALM is given by

$$T(a,b,c) = (M(I+b)a, Mb^2 + N, Mbc + NcF + P).$$

The optimal REE is a fixed point of $T(a,b,c)$. If

$$d/d\tau(a,b,c) = T(a,b,c) - (a,b,c)$$

is locally asymptotically stable at the REE it is said to be <u>E-stable</u>.

E-stability conditions were given in lecture 2.

E-stability governs stability under LS learning.

INSTABILITY RESULT

Proposition 2: *The fundamentals based i_t - rule leads to instability under learning for all structural parameter values.*

Partial Intuition: Fix all PLM parameters except a_π. Then

$$\Delta T_{a_\pi}(a_\pi) = (\beta + \lambda\varphi)\Delta a_\pi$$

via IS,PC. This tends to destabilize if $\beta + \lambda\varphi > 1$.

Conclusion: The fundamentals based reaction function can lead to indeterminacy and it always leads to instability under learning of the optimal REE.

Question: Is there an alternative interest rate setting rule that guarantees determinacy and stability?

ALTERNATIVE INFORMATION ASSUMPTION

- The instability result just stated was based on our "main information assumption": $E_t^* y_{t+1}$ depends on y_{t-1} and v_t but not on y_t.

- Under the "alternative information assumption" we permit $E_t^* y_{t+1}$ to depend on y_t and v_t (so that y_t and $E_t^* y_{t+1}$ are simultaneously determined).

- Under RE these information assumptions are the same, but they are distinct under learning.

Proposition 3: *Under the alternative information assumption and the fundamentals based reaction function there are parameter regions in which the model is stable under learning and other parameter regions in which it is unstable under learning.*

AN EXPECTATIONS BASED OPTIMAL RULE

- The instability problem can be overcome if expectations of private agents are observable and policy is conditioned on them.

- To get an optimal rule of this form solve for i_t from structural equations (IS), (PC) and the optimality condition (OPT), without imposing RE.

- That is, solve

$$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t \tag{IS}$$

$$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{PC}$$

$$\lambda \pi_t = -\alpha(x_t - x_{t-1}), \text{ all } t, \tag{OPT}$$

for i_t in terms of $x_{t-1}, E_t^* x_{t+1}, E_t^* \pi_{t+1}, g_t, u_t$.

We obtain

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t,$$

where

$$\begin{aligned}
\delta_L &= \frac{-\alpha}{\varphi(\alpha + \lambda^2)}, \\
\delta_\pi &= 1 + \frac{\lambda\beta}{\varphi(\alpha + \lambda^2)}, \\
\delta_x &= \delta_g = \varphi^{-1}, \\
\delta_u &= \frac{\lambda}{\varphi(\alpha + \lambda^2)}.
\end{aligned}$$

We call this the *expectations based reaction function*, or the expectations-based optimal i_t - rule.

This derivation made no specific assumption about expectation formation.

DETERMINACY AND STABILITY

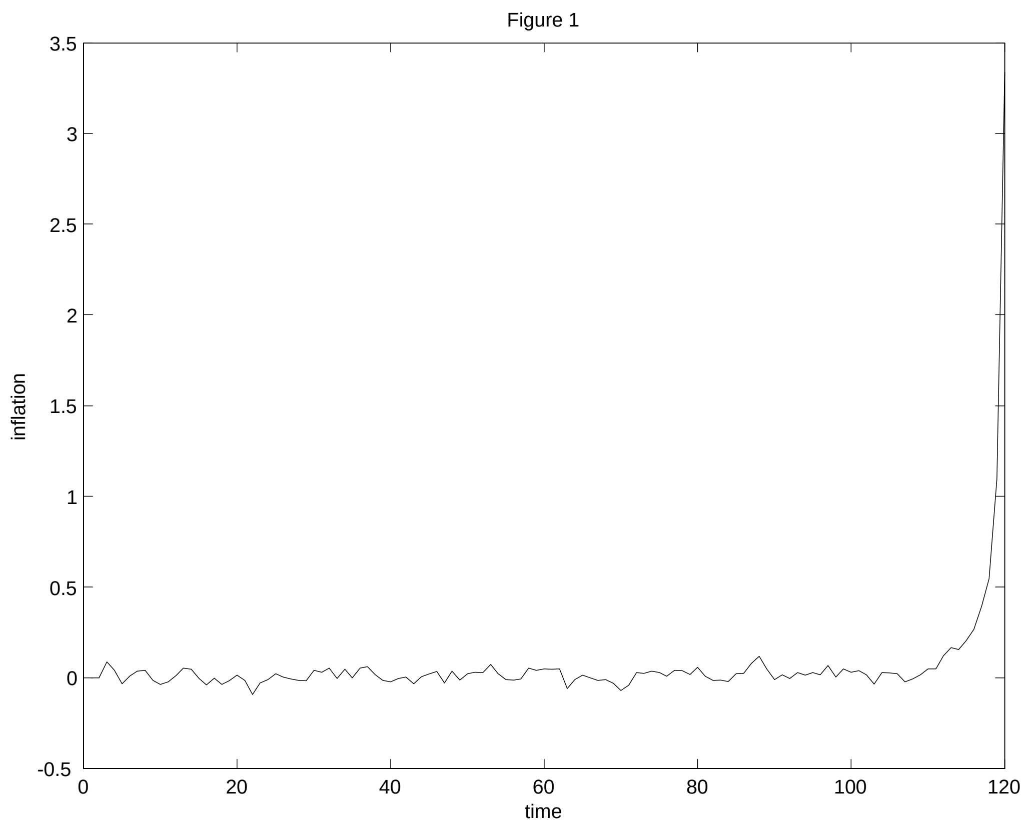
Proposition 4: *Under the expectations-based i_t - rule, the REE is determinate for all structural parameter values.*

Proposition 5: *Under the expectations-based i_t - rule, the optimal REE is stable under learning for all structural parameter values.*

Proposition 6: *The expectations-based rule is also stable under the alternative information assumption.*

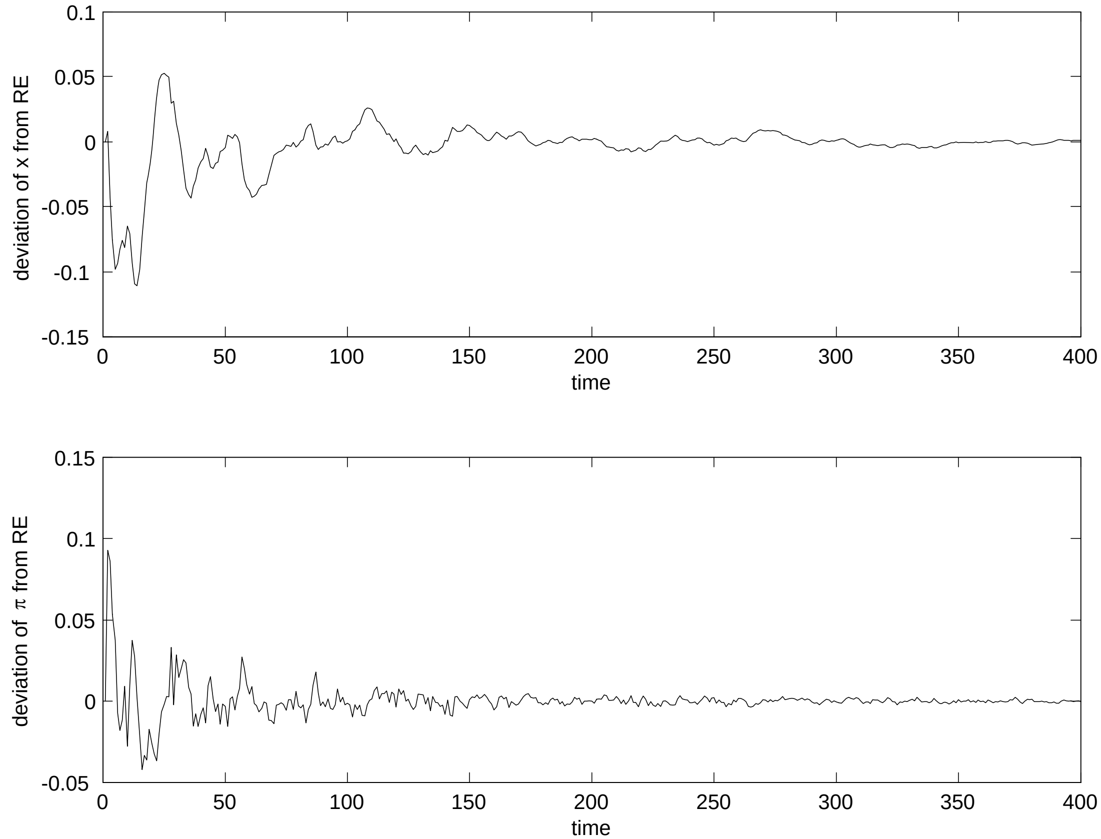
Partial intuition: $\uparrow E_t^* \pi_{t+1} \longrightarrow \uparrow\uparrow i_t \longrightarrow \downarrow x_t, \pi_t$.

Conclusion: if expectations are observable then the optimal policy can be achieved using the expectations-based i_t rule.



Instability under fundamnetals-based rule

Figure 2



Stability under expectations-based rule

Remarks:

- Stability would also hold for some variations of LS learning and even some misspecified learning schemes such as adaptive expectations

- Determinacy and stability of EB rule holds if (a) expectations observed with white noise error, or (b) VAR proxies for expectations are used. (Honkapohja and Mitra 2005a, 2006).

- We finally consider several **extensions**: (i) approximate targeting rules
(ii) structural parameter learning
(iii) robustness to structural parameter uncertainty.

EXTENSION 1: APPROXIMATING OPTIMAL POLICY

McCallum & Nelson (2001) have suggested interest rate rules that approximate

$$\lambda\pi_t = -\alpha(x_t - x_{t-1}), \tag{OPT}$$

Basic Approximate Targeting Rule:

$$i_t = \pi_t + \theta[\pi_t + (\alpha/\lambda)(x_t - x_{t-1})].$$

Determinacy and stability: Numerical results indicate that for all α, θ the REE is determinate and stable under learning.

MN show the REE is close to optimal for θ large.

MN propose alternative formulations that do not require observations of contemporaneous x_t and π_t.

Variants of Approximate Targeting Rule

MN recommend forward looking variants, e.g.

$$i_t = \tilde{E}_t \pi_{t+1} + \theta[\tilde{E}_t \pi_{t+1} + (\alpha/\lambda)(\tilde{E}_t x_{t+1} - \tilde{E}_t x_t)],$$

where $\tilde{E}_t$ denotes CB forecasts. Assume $\tilde{E}_t = E_t^*$, i.e. both CB and private agents follow LS learning.

Table 1. Indeterminacy for $\theta \geq \hat{\theta}$

α	0.1	0.5	1.0	2.0
W	0.019	0.004	0.002	0.001
CGG	1.223	0.288	0.147	0.075
MN	7.460	1.755	0.895	0.453

Table 2. Instability for $\theta \geq \tilde{\theta}$

α	0.1	0.5	1.0	2.0
W	0.038	0.008	0.004	0.002
CGG	11.61	0.704	0.324	0.156
MN	70.78	4.30	1.98	0.950

These results are a problem since low θ lead to large deviations from optimal REE.

EXTENSION 2: ESTIMATION OF STRUCTURAL PARAMETERS

- So far we've assumed φ, λ are known.

- Suppose now that policy makers estimate φ, λ :

$$\begin{aligned} w_{x,t} &= -\varphi r_t + e_{x,t} \text{ and} \\ w_{\pi,t} &= \lambda x_t + e_{\pi,t} \end{aligned}$$

where

$$\begin{aligned} w_{x,t} &= x_t - E_t^* x_{t+1} - g_t \\ w_{\pi,t} &= \pi_t - \beta E_t^* \pi_{t+1} - u_t \\ r_t &= i_t - E_t^* \pi_{t+1}. \end{aligned}$$

using recursive IV estimators.

Estimation of Structural Parameters (plus private agent learning)

We are thus now assuming:

- Private agents forecast using estimated VARs, updated by recursive LS

- Central Bank (CB) uses EB rule with estimated $\hat{\varphi}_t, \hat{\lambda}_t$, updated using recursive IV

- In EH (JMCB, 2003) we that under this **simultaneous learning** the optimal REE continues to be **locally stable**.

EXTENSION 3: ROBUSTNESS TO STRUCTURAL PARAMETER UNCERTAINTY

Evans and McGough (JMCB, 2007) consider the issue of structural parameter uncertainty further.

There is a wide range of values for ϕ, λ used in calibrations.

Name	ϕ	λ
Woodford	$1/.157$	.024
Clarida-Gali-Gertler	4	.075
McCallum-Nelson	.164	.3
variant	$1/.157$	.3

In addition there is the issue of inertia in PC and IS equations. We thus consider

$$
\begin{aligned}
IS &: \; x_t = -\phi(i_t - E_t\pi_{t+1}) + \delta E_t x_{t+1} + (1-\delta)x_{t-1} + g_t \\
PC &: \; \pi_t = \beta(\gamma E_t\pi_{t+1} + (1-\gamma)\pi_{t-1}) + \lambda x_t + u_t.
\end{aligned}
$$

Estimates of δ and γ have included values close to 1 (purely forward-looking) and 0.5. Some models have even lower δ.

To study the impact of uncertainty on the problems of stability and indeterminacy, **suppose the CB believes in a particular calibration**. We then examine one of the Taylor-type class of rules

$$
\begin{aligned}
\text{Contemporaneous:} &\quad i_t = \alpha_\pi E_t\pi_t + \alpha_x E_t x_t \\
\text{Lagged Data:} &\quad i_t = \alpha_\pi \pi_{t-1} + \alpha_x x_{t-1} \\
\text{Forward Expectations:} &\quad i_t = \alpha_\pi E_t\pi_{t+1} + \alpha_x E_t x_{t+1}.
\end{aligned}
$$

Choose the parameters α_π and α_x to solve (numerically) the optimal policy problem

$$\min_{\alpha_x, \alpha_\pi} \psi Var(x|\alpha) + Var(\pi|\alpha).$$

for that calibration. We also **impose** that the **optimized policy** deliver an REE that is **determinate & stable** under learning.

Question: can such a policy result in **indeterminacy or learning instability under a different calibration?**

Answer: yes. For either the lagged data rule or the forward expectations rule the W, MN or CGG calibrations can lead to stable indeterminacy, unstable indeterminacy or explosive behavior under some other calibrations.

Thus structural parameter uncertainty is potentially a big problem. We recommend the following procedure.

Robust i_t rules: "optimal constrained" rules

Start with a class of i_t rules parameterized by $\xi \in X$.

Structural parameters are denoted $\omega \in S$. Assume a "prior" probability distribution over their values with support $\bar{S} \subset S$.

Determine the set $P \subset X$ defined by

$$P = \left\{ \xi \in X : \text{determinacy and learning stability hold for all } \omega \in \bar{S} \right\}.$$

Assume P is non-empty (otherwise a tolerance must be specified).

Then, given a policy loss function $\mathcal{L}(\xi, \omega)$ that is well-defined for all $\xi \in P$ and $\omega \in \bar{S}$, the robust optimal constrained policy is defined by

$$\bar{\xi} = \arg\min_{\xi \in P} E\mathcal{L}(\xi, \omega),$$

where the expectation is taken over the *prior* distribution on $\bar{S}$.

To implement this we consider the class of i_t rules

$$i_t = \theta i_{t-1} + \alpha^f E_t y_{t+1} + \alpha^c E_t y_t + \alpha^L y_{t-1} + \alpha^{\hat{g}} \hat{g}_t.$$

We specify probability weights at 0.3 for W, MN and CGG and 0.1 for the variant calibration.

For the inertial parameters we set the conditional probability weights at 0.4 for $\gamma = \delta = 0.5$, at 0.2 each for $\gamma = \delta = 0.75$ and $\gamma = \delta = 0.25$, and at 0.1 each for $\gamma = \delta = 1$ and $\gamma = \delta = 0.01$.

Main findings:

- The set P was non-empty. Thus rules exist that are robust to parameter uncertainty.

- A fairly simple robust nearly-optimal was

$$\Delta i_t = E_t \pi_{t+1} + 2(E_t \pi_{t+1} - \pi_{t-1}) + 0.6 E_t x_t + 4.5(E_t x_t - x_{t-1}) + 4u_t.$$

Thus optimal policy can be designed to deliver determinacy and stability under learning in the face of structural parameter uncertainty.

CONCLUSIONS

- Optimal monetary policy design, under commitment, should not simply assume RE.

- The economy will diverge under private agent learning if the fundamentals based i_t rule is followed. Indeterminacy may also arise.

- Under our expectations-based i_t rule the optimal REE is always stable under learning, and indeterminacies are avoided.

- This policy is even locally stable under two-sided learning when CB is estimating structural parameters.

- If there is a high degree of uncertainty about structural parameters, the CB should follow "optimal constrained" rules, designed to be optimal subject to always delivering determinacy and stability under learning.

General point: Monetary policy must treat expectations as subject to shocks and be designed to be stable under learning.

ADDENDUM: PRICE LEVEL FORMULATION OF FUNDAMENTALS BASED REACTION FUNCTION

The optimization condition

$$\lambda(p_t - p_{t-1}) = -\alpha(x_t - x_{t-1}) \tag{OPT}$$

can be rewritten as

$$x_t = -\frac{\lambda}{\alpha}p_t + k, \text{ for any } k. \tag{OPT'}$$

This yields an REE

$$\begin{aligned} p_t &= \bar{b}_x p_{t-1} + \bar{c}_p u_t + \bar{a}_p \\ x_t &= \bar{b}_p p_{t-1} + \bar{c}_x u_t + \bar{a}_x \end{aligned}$$

with alternative fundamentals based reaction function

$$i_t = \eta_p p_{t-1} + \varphi^{-1} g_t + \eta_u u_t + \eta_0.$$

Result: The optimal REE is unstable under learning if $\varphi > \lambda/\alpha$.