Learning and Monetary Policy

Lecture 2 – The New Keynesian Model of Monetary Policy: Determinacy and Stability under Learning

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Outline

- We study the linearized (multivariate) NK (New Keynesian) model, with various Taylor-type interest-rate rules.

- Depending on the form and parameters of the i_t rule, there are (separate) issues of
  (i) determinacy (uniqueness) and (ii) stability under learning

- Determinacy and stability are analyzed for different i_t-rules.

- For cases of indeterminacy we also look at whether sunspot equilibria can be stable under learning.

The New Keynesian Model

- Log-linearized New Keynesian model (Clarida, Gali and Gertler 1999 and Woodford 2003 etc.).

  1. "IS" curve

  $$x_t = -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t$$

  2. the "New Phillips" curve

  $$\pi_t = \lambda x_t + \beta E_t^* \pi_{t+1} + u_t,$$

where $x_t =$ output gap, $\pi_t =$inflation, $i_t =$ nominal interest rate. $E_t^* x_{t+1}$, $E_t^* \pi_{t+1}$ are expectations. Parameters $\varphi, \lambda > 0$ and $0 < \beta < 1$.

- Observable shocks follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \; F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

where $0 < |\mu|, |\rho| < 1$, and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$.

- Some versions of the NK model incorporate inertia, i.e. x_{t-1} in the IS curve (due to habit persistence) and π_{t-1} in the PC curve (e.g. use of indexation by non-optimizing price-setters).

Monetary Policy Rules

To complete the model we add a policy rule for i_t. Interest rate setting by a standard **Taylor-type instrument rule**

$$i_t = \pi_t + 0.5(\pi_t - \bar{\pi}) + 0.5x_t,$$

where $\bar{\pi}$ = target inflation rate and target x_t is zero.

More generally (with $\bar{\pi} = 0$)

$$i_t = \chi_\pi \pi_t + \chi_x x_t, \text{ where } \chi_\pi, \chi_x > 0.$$

Variations: replace π_t, x_t by lagged or expected future values, e.g.

$$i_t = \chi_\pi \pi_{t-1} + \chi_x x_{t-1}, \text{ or}$$

$$i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}$$

A number of variations of the rule have been studied, e.g. with interest-rate smoothing,

$$i_t = \theta i_{t-1} + \alpha_\pi \pi_t + \zeta_x x_t.$$

In this lecture we focus on the conditions for determinacy (uniqueness) and stability of the REE under learning for various interest-rate rules.

Determinacy and Stability under Learning

MULTIVARIATE LINEAR MODELS

Combining IS, PC and the i_t rule leads to a bivariate reduced form in x_t and π_t. Letting $y_t' = (x_t, \pi_t)'$ and $v_t' = (g_t, u_t)'$ the model can be written

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = M \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + N \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \end{pmatrix} + P \begin{pmatrix} g_t \\ u_t \end{pmatrix},$$

$$\begin{aligned} y_t &= M E_t^* y_{t+1} + N y_{t-1} + P v_t. \\ v_t &= F v_{t-1} + \tilde{v}_t. \end{aligned}$$

We consider general linear models for vectors of endogenous y_t and exogenous v_t. In some cases we have $N = 0$ and thus

$$y_t = M E_t^* y_{t+1} + P v_t.$$

SIMPLE MODEL WITHOUT LAGGED y_{t-1}

Consider first the simpler model

$$\begin{aligned} y_t &= M E_t^* y_{t+1} + P v_t, \\ v_t &= F v_{t-1} + \tilde{v}_t, \end{aligned}$$

where $\tilde{v}_t' = (\tilde{g}_t, \tilde{u}_t)$.

Condition for determinacy: all roots of M lie inside the unit circle.

If the model is determinate the REE takes the form

$$y_t = \bar{c} v_t$$

The solution is obtained from

$$\begin{aligned} E_t y_{t+1} &= \bar{c} F v_t, \text{ and } \bar{c} v_t = M \bar{c} F v_t + P v_t, \\ &\longrightarrow \bar{c} = M \bar{c} F + P. \end{aligned}$$

LEARNING IN THE SIMPLE MODEL

Replace RE by LS learning. Agents estimate

$$y_t = a + cv_t.$$

Estimates a_t, c_t are updated using LS regressions of y_t on v_t and an intercept.

Expectations are given by

$$E_t^* y_{t+1} = a_t + c_t F v_t.$$

The question: over time does

$$(a_t, c_t) \rightarrow (0, \bar{c})?$$

Convergence to an REE depends on "expectational stability" (or "E-stability") conditions.

LS LEARNING: E-STABILITY METHODOLOGY

Reduced form

$$y_t = ME_t^* y_{t+1} + Pv_t.$$

Stability under learning is analyzed using E-stability:

Under the PLM (Perceived Law of Motion)

$$\begin{aligned} y_t &= a + cv_t \\ E_t^* y_{t+1} &= a + cFv_t. \end{aligned}$$

This $\longrightarrow$ ALM (Actual Law of Motion)

$$y_t = Ma + (P + McF)v_t.$$

Mapping from PLM to ALM

$$T(a, c) = (Ma, P + McF).$$

We have:

$$T(a, c) = (Ma, P + McF).$$

The optimal REE is a fixed point of $T(a, c)$. If

$$d/d\tau(a, c) = T(a, c) - (a, c)$$

is locally asymptotically stable at the REE it is said to be E-stable.

E-stability conditions: For the model these are:

(i) all eigenvalues of M have real parts less than 1
(ii) all products of eigenvalues of M with eigenvalues of F have real parts less than 1.

E-stability governs stability under LS learning.

For the simple model determinacy $\Rightarrow$ E-stability

THE GENERAL MODEL

Now consider the model with lagged y_{t-1}

$$y_t = ME_t^* y_{t+1} + Ny_{t-1} + Pv_t.$$

Determinacy: There are standard techniques for checking determinacy, i.e. uniqueness of REE.
If the model is "determinate" there exists a unique stationary REE of the (MSV) form

$$\begin{aligned} y_t &= \bar{b}y_{t-1} + \bar{c}v_t \text{ if } N \neq 0 \text{ or} \\ y_t &= \bar{c}v_t \text{ if } N = 0. \end{aligned}$$

If "indeterminate" there are multiple solutions. These include multiple MSV solutions and also other (undesirable) stationary sunspot solutions.

Determinacy condition: compare # of stable eigenvalues of matrix of stacked first-order system to # of predetermined variables.

LS LEARNING

Under learning, agents have beliefs or a perceived law of motion (PLM)

$$y_t = a + by_{t-1} + cv_t,$$

and estimate (a_t, b_t, c_t) in period t based on past data.

- Forecasts are computed from the estimated PLM.
- New data is generated according to the model with the given forecasts.
- Estimates are updated to $(a_{t+1}, b_{t+1}, c_{t+1})$ using least squares.
- Question: when is it the case that

$$(a_t, b_t, c_t) \rightarrow (0, \bar{b}, \bar{c})?$$

E-STABILITY

Reduced form

$$y_t = ME_t^* y_{t+1} + N y_{t-1} + P v_t.$$

Stability under learning is analyzed using E-stability:

Under the PLM (Perceived Law of Motion)

$$y_t = a + b y_{t-1} + c v_t.$$

$$\begin{aligned} E_t^* y_{t+1} &= a + bE_t^* y_t + cF v_t = a + b(a + bE_t^* y_t + cF v_t) + cF v_t, \text{ or} \\ E_t^* y_{t+1} &= (I + b)a + b^2 y_{t-1} + (bc + cF) v_t. \end{aligned}$$

This $\longrightarrow$ ALM (Actual Law of Motion)

$$y_t = M(I + b)a + (Mb^2 + N) y_{t-1} + (Mbc + NcF + P) v_t.$$

Remark: This assumes the time t **information set** is: $I_t = \{v_t, y_{t-1}, v_{t-1}, \ldots\}$. The condition is somewhat different when the information set also includes y_t . See Evans & Honkapohja (2001), Ch. 10.

With this **"alternative" information assumtion**, expectations are given by

$$E_t^* y_{t+1} = a + by_t + cFv_t,$$

and the ALM is

$$y_t = (I - Mb)^{-1} Ma + (I - Mb)^{-1} McFv_t + (I - Mb)^{-1} Ny_{t-1} + (I - Mb)^{-1} Pv_t,$$

whereas with the **"standard" information assumption** we have the ALM

$$y_t = M(I + b)a + (Mb^2 + N)y_{t-1} + (Mbc + NcF + P)v_t.$$

We proceed using the "standard" information assumption, but it is easy to work out the other case too.

The ALM gives a **mapping from PLM to ALM:**

$$T(a,b,c) = (M(I+b)a, Mb^2 + N, Mbc + NcF + P).$$

The optimal REE is a fixed point of $T(a,b,c)$. If

$$d/d\tau(a,b,c) = T(a,b,c) - (a,b,c)$$

is locally asymptotically stable at the REE it is said to be **E-stable**. See EH, Chapter 10, for details. The **E-stability conditions** can be stated in terms of the derivative matrices

$$\begin{aligned} DT_a &= M(I+\bar{b}) \\ DT_b &= \bar{b}' \otimes M + I \otimes M\bar{b} \\ DT_c &= F' \otimes M + I \otimes M\bar{b}, \end{aligned}$$

where $\otimes$ denotes the Kronecker product and $\bar{b}$ denotes the REE value of b.

E-stability governs stability under LS learning.

Results for Interest Rate Rules

We now apply these techniques to the Taylor-type interest-rate rules considered by Bullard and Mitra.

CONTEMPORANEOUS DATA RULE

For the rule

$$i_t = \chi_\pi \pi_t + \chi_x x_t$$

we get the simple model

$$y_t = M E_t^* y_{t+1} + P v_t,$$

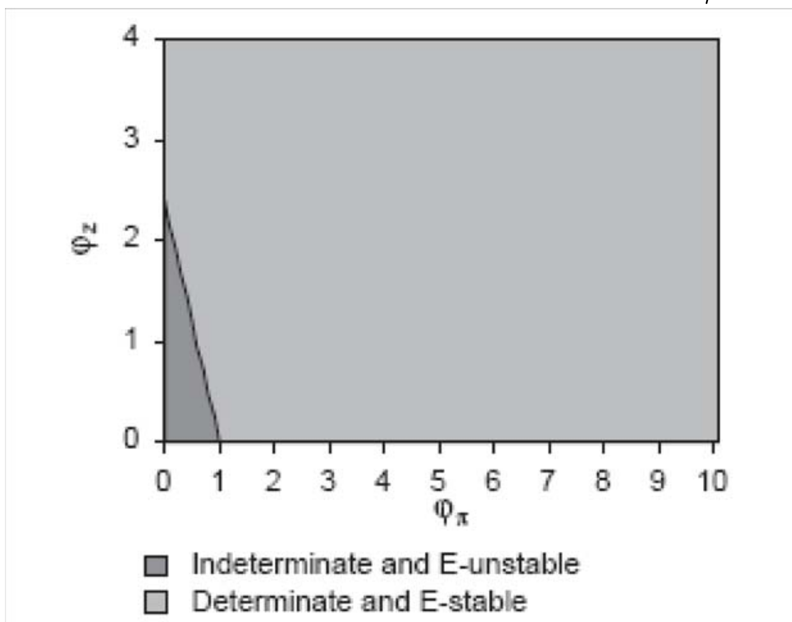
with

$$M = \frac{1}{\phi^{-1} + \chi_x + \lambda \chi_\pi} \begin{pmatrix} \phi^{-1} & (1 - \beta \chi_\pi) \\ \lambda \phi^{-1} & \lambda + \beta(\phi^{-1} + \chi_x) \end{pmatrix}.$$

B&M show that the condition

$$\lambda(\chi_\pi - 1) + (1 - \beta)\chi_x > 0.$$

is necessary and sufficient for both determinacy and E-stability. The Figure shows the results for the "Woodford" calibration of ϕ and λ.



(B&M use φ_x, φ_π for χ_x, χ_π). The "Taylor principle" $\chi_\pi > 1$ is sufficient.

LAGGED DATA RULE

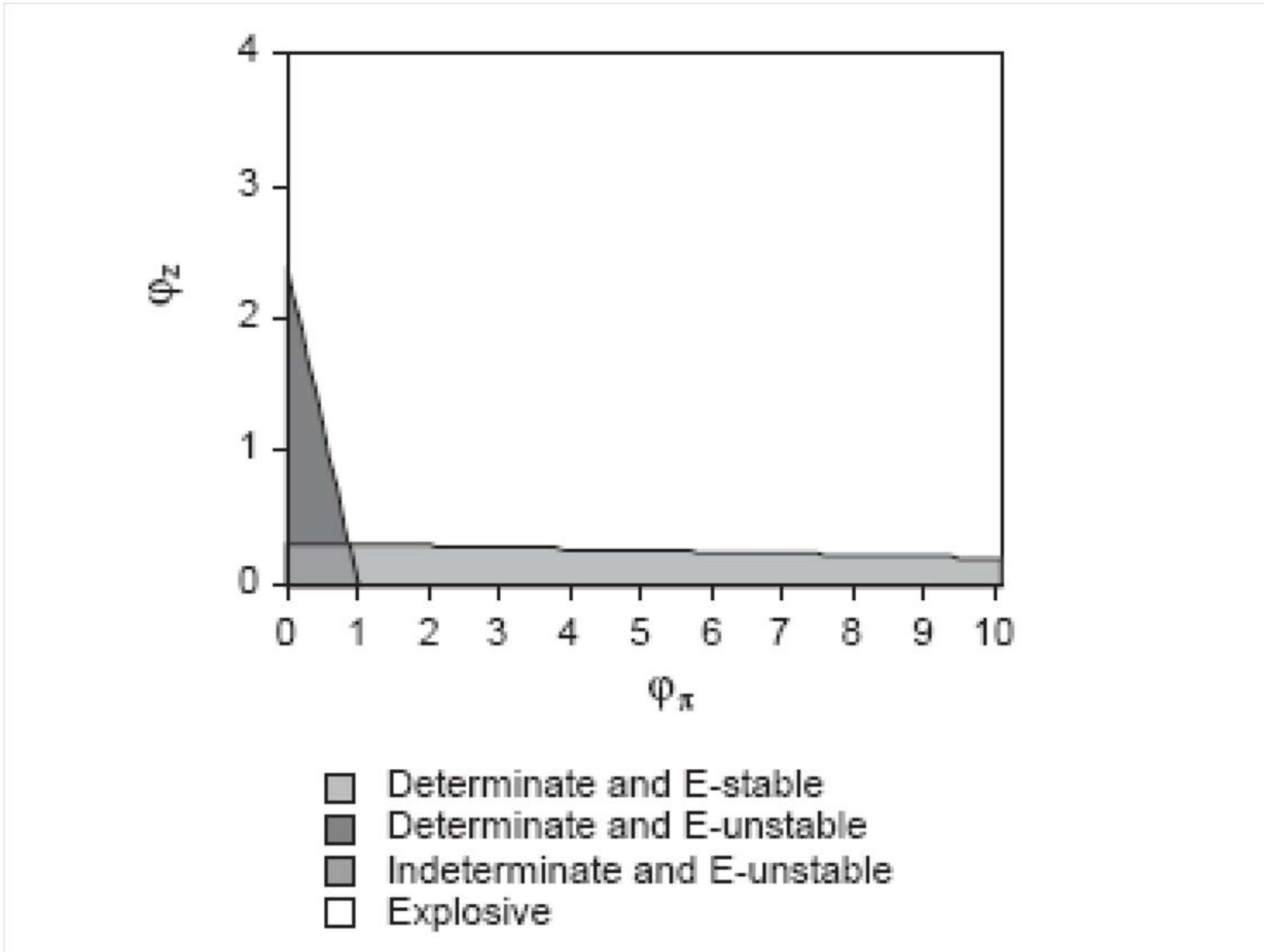
For the rule

$$i_t = \chi_\pi \pi_{t-1} + \chi_x x_{t-1}$$

we can solve for M, N in the general framework.

The determinacy and E-stability conditions are now more complex.

Note that determinate but E-unstable is possible.



Lagged data rule

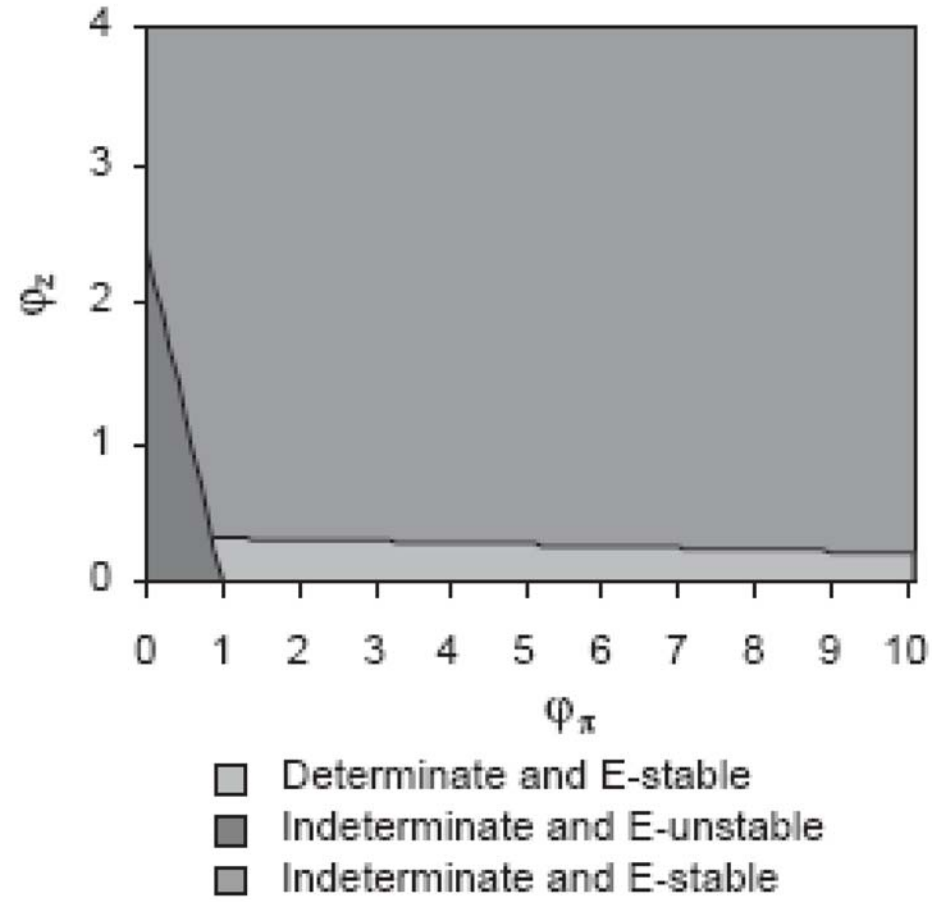
FORECAST BASED ("FORWARD-LOOKING") RULE

For the rule

$$i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}$$

we are again back in the simple framework.

The main additional case of interest is that for $\chi_\pi > 1$ but $\chi_x > 0$ too large, it is possible for to have Indeterminate but E-stability of one of the MSV solutions. We return to this below.



Forecast-based rule.

Further points.

- Non-observability of current variables y_t in contemporaneous data rule.

  B&M show that using $E_t^* y_t$ or $E_{t-1}^* y_t$ in place of y_t does not affect determinacy and E-stability.

- Interpretation of expectations in forecast-based rules

$$i_t = \chi_\pi E_t^* \pi_{t+1} + \chi_x E_t^* x_{t+1}.$$

  Analysis assumes homogeneous expectations for private sector and CB: (i) CB bases i_t on private sector forecasts or (ii) private sector uses CB forecasts, or (iii) private sector and CB forecast in the same way, using VARs.

- If CB bases i_t on private sector forecasts, but there are white noise measurement errors, E-stability and determinacy are unaffected.

- CB can use internal VAR forecasts as a proxy. E-stability is unaffected even though CB and private sector can have different VAR forecasts, due to different priors.

Learning Sunspots

- Clarida, Gali and Gertler (2000) argued that in the US:
  (i) The US had a forward-looking Taylor rule
  (ii) Pre-1980 the long-run coefficient χ_π on $E_t^* \pi_{t+1}$ was $\chi_\pi < 1$, consistent with indeterminacy and SSEs, while after 1980 $\chi_\pi > 1$ and the model was determinate.

- Can SSEs (stationary sunspot equilibria) be stable under learning in NK models? This is discussed in Honkapohja and Mitra (2004), Evans and McGough (2005, 2006), Eusepi 2007.

- In the simple model without lags the MSV solution is $y_t = \bar{c} v_t$. In the indeterminate case there are SSEs taking the form

$$y_t = \bar{c} v_t + \bar{d} \zeta_t,$$

where ζ_t can be either a k-state sunspot with suitable transition probabilities or an exogenous AR(1) sunspot with suitable damping coefficient.

- Under LS learning will an SSE be stable? E-stability conditions can be computed. Evans and McGough (2005) find the following:

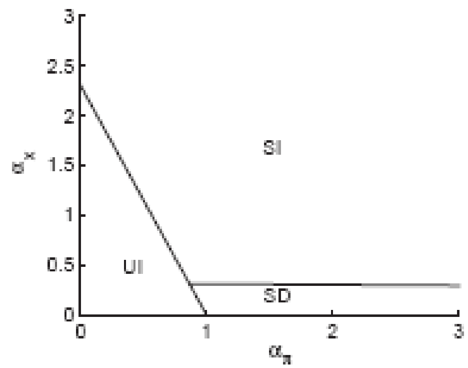


Fig. 4. PR$_3$, W cal., $\gamma = 1$.

There are two indeterminacy regimes. SI (stable sunspots) do exist, but for large χ_π and χ_x too large.

- Interest-rate smoothing helps increase the area of stable determinacy. The CGG rule takes the form

$$i_t = \theta i_{t-1} + \alpha_\pi E_t^* \pi_{t+1} + \zeta_x x_t,$$

and they find $\theta = 0.68$. For the CGG calibration we have

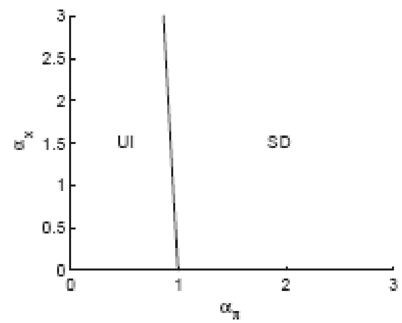


Fig. 7. PR_4, CGG cal., $\gamma = 1$, $\theta = .68$.

For the estimated CGG pre-1980 rule we are in the UI region, i.e. the SSEs are unstable under learning.

- Stable sunspots also exist in market-clearing seigniorage models. See Evans, Honkapohja, Marimon (2007).

Conclusions

- Stability under learning can be analyzed in multivariate linearized models like the standard NK model.
  (i) Like determinacy, learning stability of an REE can be checked from eigenvalues of suitable matrices.
  (ii) Determinacy and learning stability are distinct conditions.

- Results for Taylor-type i_t rules:
  (i) Contemporaneous data Taylor-rules are determinate and stable under learning for suitable parameter values.
  (ii) Forward-looking rules are determinate, stable for appropriate parameter choices. For others, stable or unstable indeterminacy can arise.

- Next lecture: what about optimal policy?

Addendum:

E-stability under alternative information assumption in which y_t is included in the information set of agents when making forecasts.

The E-stability condition in this case is that is that all the eigenvalues of

$$\begin{aligned} DT_a - I &= (I - M\bar{b})^{-1}M - I, \\ DT_b - I &= [(I - M\bar{b})^{-1}N]' \otimes [(I - M\bar{b})^{-1}M] - I, \\ DT_c - I &= F' \otimes [(I - M\bar{b})^{-1}M] - I \end{aligned}$$